

A READING ARTICLE

A significant percentage of students find mathematics-related courses intimidating, enough so that the term “math anxiety” became part of the academic and popular lexicons some 25 years ago (Tobias, 1980). And even those who demonstrate facility in manipulating quantitative information in its own environment, i.e. without application, often recoil at the sight of the dreaded “word problems.” This reaction begins early and continues through college. It reaches its apex when students, even those academically oriented enough to pursue a graduate degree, have major difficulties with complex verbally presented problems or cases in such courses as statistics or econometrics.

A clue to their issues lies outside the quantitative realm as students also struggle with courses such as organizational behavior and strategy when it comes to analysis of cases. Fundamentally, reading and analyzing information that cannot be held or expressed as a single mental “unit,” i.e. one that requires a process, often confounds them. I suggest here that students do not generally have adequate training in reading with reflection while instructors typically assume they have picked this up along the way. I build a model for reading in the context of solving complex verbal problems in mathematics based courses, with the hope that faculty in other disciplines will find it applicable there as well. I then go on to provide a plan for learning how to read more effectively.

A Model for Reading

Drawing on the research of Scardamalia and Bereiter (1991), Schoenfeld (1989), and Sankowsky (2003, 2004) on reading and problem solving, in addition to the framework of cognitive psychology (Bedrosian & Beck, 1980; Hornby, 1990), I link *anticipation* (perception of an event and expectations about addressing it) with *affect* and *activities*, the latter referring to various mental maneuvers as well as overt behaviors. When the event is a complex verbal problem in a quantitative methods course and addressing it means trying to find a solution, students’ engagement with it requires them to read, translate verbal information into symbolic code, and compute.

Focusing solely on reading in this article, I categorize these activities as the 7A inputs – *attending, accumulating, assuming, associating, asking, analyzing, and acting* – and the 2A outcomes, *assessing* and *attributing*, elaborations forthcoming. Each of these has a bipolar continuum: for example, a student may make an assumption (the assuming activity) about how to approach a problem, responding in an open manner to instructor feedback or, on the other hand, responding with a defensive posture (or any place in between). The 7A hopefully produce “an answer,” triggering the 2A outcome activities. (The

separation between input and outcome activities is not so clear cut as I will explain).

Attending refers to the way a student absorbs/focuses on the information as s/he encounters it. The related continuum goes from “*following the flow*,” having almost every individual information fragment make sense on its own, producing the overall feeling of “comprehending,” to not following, having an overall sense of being lost and confused, either by the totality of information or by even a single fragment.

Accumulating refers to the way a student stores the information. The related continuum goes from *globally flexible*, marked by a sense of the emergence of a “big picture” with the confidence that one can retrieve necessary details when the time comes, to *locally inflexible*, marked by the pressure to hold all the information in one’s head, including the details.

Associating refers to way the student uses prior experience, especially in discipline. The related continuum goes from multiple and micro, meaning that he or she seeks to make connections with different aspects of several other problems and not necessarily within the same context, to single macro, meaning that he or she seeks to make a total connection with another problem, generally in the same context.

Assuming refers to the way a student takes certain aspects of a situation as given – the nature of the assumptions he or she makes. The related continuum goes from explicit and testable assumptions to tacit and conclusive ones.

Asking refers to the way and extent to which a student probes the given information. The related continuum goes from critical reflection, asking the self various routine questions while using a repertoire of prompts or requesting clarification from others, to the lack of such reflection -- in part because of the absence of a helpful repertoire of questions and prompts, and also because of the perceived intimidation of asking another person to clarify (possibly secretly hoping that some individual will actually step in do the problem).

Analyzing refers to the reasoning process as the student moves from the information given toward a solution direction. The related continuum goes from trained inquiries based on the discipline in order to link concepts with contexts as a basis for subsequent reasoning to undisciplined private or naïve reasoning.

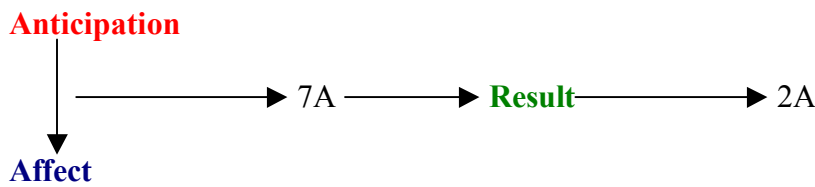
Acting refers to a visible behavioral product, as in generating an answer or a solution direction. It definitely occurs at the end of the process (unless the process is aborted), but can take place at any time along the way. The related

continuum ranges from appropriate, when the student is ready to express an idea publicly, to premature, when he or she feels pressure – usually self-imposed – to do something at a certain point.

When students complete a reading cycle, they may have an answer or a solution direction at the ready. If they do not, they have the option of cycle reiteration, i.e. rereading the problem with the same range of activities. They may choose to do so several times, particularly if they find the information very dense or complex and also if they have trouble with accumulating, i.e. with keeping it under some cumulative control.

When they do produce an answer, they then receive feedback. At that point, they engage in the activity of (self) *assessing*, with a continuum ranging from accepting the challenge of making necessary changes to rejecting the feedback/giving up because of it. We assume in this scenario that their performance does need some improvement, but not necessarily that it is substandard. They then engage in *attributing*, with a continuum ranging from questioning any assumptions that may have led them astray in a constructive manner to playing the blame game, impugning instructors, texts, classmates, and ultimately themselves for a perceived failure.

FIGURE ONE



The activities themselves as a group range from productive/open to self-defeating/closed. I have set up the poles for each activity with the first one productive and the second one self-defeating. Similarly, anticipation has a continuum, ranging from the perception of the problem as an interesting challenge to seeing it as an overwhelming burden. And affect ranges from positive with eager curiosity and confidence to negative with a feeling of intimidation/fear.

FIGURE TWO

ACTIVITY	POSITIVE	NEGATIVE
Anticipation	Challenge	Burden
Affect	Confident	Overwhelmed
Attending	Following line by line	Not following
Accumulating	Global flexible	Local inflexible
Associating	Multiple & micro	Single macro
Assuming	Explicit & testable	Tacit & conclusive
Asking	Reflection w/repertoire	Lack of reflection
Analyzing	Trained inquiries	Private reasoning
Acting	When appropriate	Premature
Assessing	Accept feedback	Reject feedback
Attributing	Question assumptions	Blame; lack of ability

Characteristics of the activities

Typically, attending and accumulating occur nearly simultaneously throughout the reading enterprise. The attending activity is often considered as reading per se, since it features the individual's real time encounter with the written information. As a separate activity, accumulating also has a major role to play since it concerns itself with the way the student relates information currently encountered to information previously received.

Then the other activities *punctuate* the process. That means they occur sporadically, interjected into the reading enterprise and interrupting the attending and accumulating. There are two types, random and decision punctuating activities. By random, we mean beyond the control of the student while by decision, we mean that the student chooses when to punctuate in this way.

For example, associating to other problems comes about spontaneously, as the student makes a connection with them based on experience, knowledge, and the sense-making gleaned from the ongoing attending/accumulating activities. It makes its appearance and has an impact on the process in terms of the ultimate response, with only a brief time in the spotlight. Similarly, assuming occurs briefly and spontaneously, for example, about content ("what this must mean"), task ("what I must do"), and self ("what I can do"), based on the student's academic history, perception of the problem, and the ongoing attending/accumulating. Both associating and assuming are random activities.

The final three of the seven input activities are decided by the reader rather than randomly occurring. At various times, the student may choose the asking activity, with (generally self) queries about the appropriate framework, about the identity of the relevant variables, and about the problem statement. This activity leads directly to analyzing, in that prompts in the form of questions help the student recognize the link between concepts and context, an important aspect of the analysis activity. Analyzing is also at center stage at any time the student invokes reasoning. For example, in a linear programming problem, when given the information “there are twice as many economy tires as radials,” (Eppen, Gould, & Schmidt, 1996) more than half of a management science class wrote the equation $2E=R$ instead of the correct $E=2R$. The instructor queried the class to get at the reasoning producing the error. This process revealed that most students had *visualized* two groups of tires, the economy pile twice as big as the radial. They then juxtaposed the 2 and the E. Their private reasoning occurred implicitly: had students articulated their thinking, it would likely have been along the lines of “the E pile is twice as big; therefore the 2 goes with the E.” They needed to come to terms with this process in order to then fully accept the translation required to express the equation and the reasoning behind it (Sankowsky, 2003).

Both activities either interrupt the attending activity or wait until it has been completed. Similarly, acting can interrupt the attending as a decision activity when a student feels the need to externalize something – an association leading to an idea that he or she sees as important enough to write down. At times, if the idea gets the student thinking, it can flourish into a useful creative outburst. At other times, it can take students in the wrong direction as the tire example shows, even though they feel ready for action. The pattern of acting coming at the end, following attending and the other activities, is not necessarily the most common.

Assessing and attributing, the two “outcome” activities, take place primarily after the student has produced an answer. They can also occur during the reading cycle, punctuating the process, as students interrupt what they are doing to take stock of how they are doing. These are not necessarily choices for the student – rather he or she may be at the mercy of unconscious processes triggered by the information encountered.

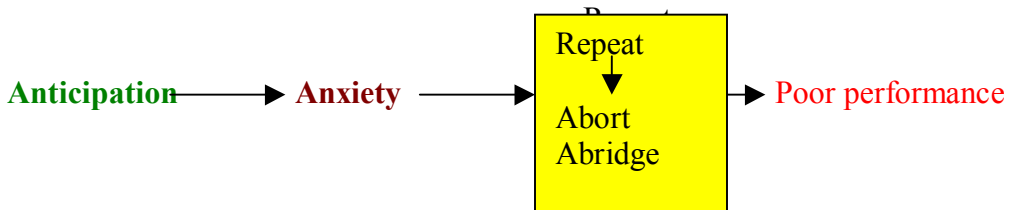
FIGURE THREE

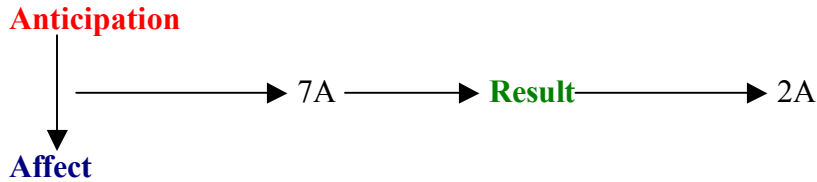
ATTENDING					
ACCUMULATING					
Associating	/		/		/
Assuming	/	/	/	/	
Asking		/		/	
Analyzing		/		/	/
Acting	/				/
Assessing		/			/
Attributing					/

Hypothetical punctuations for the activities

Not all activities appear in a given reading *cycle*: one completes a cycle when the attending activity finishes, i.e. quite simply, when one reaches the last line of the problem or case. Often it does not occur to students to engage in analysis or asking (Schoenfeld, 1989) as they *abridge* the reading process, generally to the detriment of their performance. They may also *abort*, giving up before completing the cycle. Finally, they may not avail themselves of the associating activity, because they fail to make any meaningful connections with other problems.

FIGURE FOUR





Poor Performance in Reading

Reading badly almost always undermines problem solving irretrievably. So this is where we focus our attention. Several cases dot the substandard reading landscape (elaborations forthcoming):

- Abridging, especially leaving out asking, analyzing, associating
- Living on the wrong side of the continuum for most activities
- Attending/accumulating abort
- Rereading with no goal differentiation
- Acting prematurely

Abridging has been discussed: by not taking advantage of all the activities qua resources, students truncate their opportunities to eventually discover a pathway to a solution.

On the other hand, using all the activities does not guarantee success. Living on the wrong side of the continuum refers to the fact that for each activity, as previously indicated, there are two poles, the first of which is set up as productive and the second as self-defeating. It follows that students who (involuntarily) position themselves close to the second pole in many of the input activities are likely to produce substandard results.

The last three cases involve interactions between the 7A input activities. The attending/accumulating abort occurs when students encounter a piece of information they cannot grasp and then assume that they cannot progress without knowing what the detail means – or when they find the totality of details so perplexing that they become overwhelmed to the point of giving up. For example, students trying to represent a problem with a decision tree came up against the following sentence: “JM, a small manufacturer of metal parts, was considering whether or not to enter the competition to be a supplier of transmission housing for PROTRAC.” Some became fixated on the term “transmission housings” and felt they could go no further without knowing what that meant. Some others simply went directly to that feeling, unaware of the assumption underlying it.

In the second case, the student goes through several cycles, without (necessarily) abridging and not necessarily on the wrong end of the continuum. The student completes the reading cycle with no breakthroughs: perhaps too much detail prevents a solution direction from emerging; perhaps the accumulating activity cannot hold enough information in storage on one pass. In any case, the student reflexively rereads the problem, hoping for greater penetration the second, and even third or fourth time through. The rereading, however, tends to be ineffective if it has the same orientation and covers the same basic ground with the same basic overall goal – get the answer. We will see a way around this shortly.

In the third case, the student punctuates the attending and accumulating activities with acting, much too often and much too early. For example, he or she reads a line and tries to write an equation or begin building a decision tree. Sankowsky (2003) offers the following example: the plight of one student working a complex problem who tried to construct a decision tree as he read. He got in trouble because he tacitly and automatically took the first of the three companies presented in the problem as the company of record, the one from whose perspective the tree is to be constructed. When he read that the first company was considering various options about where to build a new computer facility, he correctly identified that as its decision, but then went on to assume it should appear as such on the tree. However, had he read further before taking action, he would have realized that the third company named in the problem, one that purchases land, was the company of record – and the first company's computer placement decision was its uncertainty, to be represented as such on the tree.

Toward More Effective Reading

To address the five common poor performing scenarios, I propose the 3M approach, a reading system featuring multiple reads with differentiated goals (each cycle has its own), a “matrix” methodology to link concepts with contexts, a repertoire of prompts to enable this, and a meta-vocabulary, based on a structural orientation.

Rereading differentially establishes several expectations for students. First, they know going in that they will read the information more than once so accumulating it can occur in a more relaxed and flexible way, over time. Second, each cycle will focus on a particular goal, so it will have a reduced scope, thereby taking away some pressure. Third, some of the passes ignore details altogether, in effect eliminating the struggle to master them prematurely. And fourth, it dovetails with the other features of this approach, e.g. one read through may

have as its goal the linkage between concepts in the abstract and their manifestation in the context of the problem.

The matrix methodology refers to a two-prong way of forging this link: through *application*, where students start with the discipline and ask themselves, “how does concept A manifest itself in this problem?” and through *representation*, where students start with information contained in the problem and ask themselves, “what concept does information fragment B illustrate?”

Such questions generally require still more direction in the form of additional prompts for many students. Consider, for example, the identification of the decision variables as a goal for one of the reads in a linear programming setting. I suggest to my students that they use VCR if they find themselves stuck: visualize, conceptualize, and reframe. In a problem involving a grain company with two farms and three crops, they can picture two rectangles (or irregular shapes if they prefer) subdivided into 3 areas; they can conceptualize the decision as a planting/seeding/allocating of land; and finally, they can use a template to come up with “the company must decide how many acres of land to set aside for the planting of each of the crops on each of the farms.”

To help them accomplish this, still another layer of the 3M comes in handy: a meta-vocabulary. Even though instructors themselves use such terminology as “unit quantities,” “totals,” “limitations” (for less than or equal to constraints), and the like, their students frequently hear these words, but do not internalize them. Reading differentially with one pass dedicated to picking out just unit quantities and totals will reinforce students’ comfort with and command of these terms.

To further the meta-vocabulary, a structural orientation proves helpful. Structural analysis takes content of the discipline as object rather than subject (Kegan, 1994): it assumes a knowledge of the relevant constructs within the discipline and then erects *taxonomies* (typing concepts), pinpoints *distinctions*, “unpackages” *information* (e.g. breaks concepts down into more usable pieces), and reveals *transitions* (e.g. from individuals to relationships). For example, several types of constraints can be named in linear programming contexts: process, supply and demand, and interrelationship, to name a few. Types of problems can be identified. Equations for constraints can be broken down: verbal descriptor, units, totals, coefficients, and so on. Instructors must take care not to blitz students with technical terminology; instead, they need to fold it into the fabric of the problem solving so it is seen to lessen an existing burden rather than adding a new one.

This approach addresses each of the aforementioned poor reading scenarios. First, obviously, the multiple reads with differentiated goals strategy directly challenges rereading with no differentiation. Second, as indicated, knowing they will reread takes some pressure off their initial foray, so students have a vehicle that better enables them to delay acting until an appropriate time. Third, with specific but yet broad goals, there is also less reason to obsess over a single detail and become locked into the attending/accumulating abort. Fourth, since abridging generally means leaving out asking and analyzing, making explicit a repertoire of prompts that aid in recognition of the link between abstract ideas and specific examples provides the keys to those two activities. Finally, and perhaps most important, students will move toward the productive side of each continuum.

Why is this? The matrix methodology and meta-vocabulary make linking concepts and contexts easier and more explicit – especially with goal differentiation that focuses on certain links as the aim of a given read. So the analyzing activity is enhanced. Structural terminology aids associating as it frees students from context and allows them to see deeper connections between problems. The repertoire of prompts directly impacts the asking activity, provided these are made explicit as such. As indicated, students have a vehicle with which to wait and hence delay acting. Learning an explicit approach that counteracts several counterproductive tendencies challenges tacit assumptions, so the assuming activity is positively impacted. Attending and accumulating are enhanced by having a better sense of structure and by the reduction in pressure to grasp and hold information from anticipating several (goal differentiated) read throughs.

On the outcome side, the multiple reads tend to allow for more positive assessing in general and more tolerance when progress is slow in particular: interim goals are more accessible than finding an overall solution and they promote a sense of process that allows students to refrain from premature judgment. And should the reading enterprise founder, attributing that to lack of innate ability is less likely than attributing it to not following the system – i.e. there is now something definite to work with even should results not instantly change for the better.

An Illustration

Each read through can be broken down further, into two passes, with one a scan and the other a line by line parsing of information. The first consists of the application side, with the second the representation side.

For example, consider the reading stages recommended for linear programming, each with its own specific goal. In the first reading, students wade through the

details seeking to identify the variables and the objective (function). To do so, they will benefit from various prompts such as asking themselves, “What is the primary action that is to be taken by the decision maker?” They are encouraged to visualize what is going on and determine what the problem is asking the decision maker to do. Sometimes, as indicated earlier, a template will help, e.g. (fill in the blanks): they have to decide how many/much of each (type of) ___ to ___ in order to max/min ___. In the second reading, the goal is to make a complete list of variables and constraints the latter incorporating the internal and external environment into the decision in the form of limitations and requirements. Armed with a constraint taxonomy, students are in a better position to avoid the trap of prematurely dealing with detail, as they concentrate instead on categorization. The goal of the third reading of a linear programming problem is to identify numeric information using structural terminology such as “unit quantities” and “totals,” and linking these numbers to specific variables and constraints, while paying special attention to the units.

In more detail, the reading stages recommended for linear programming along with their goals and prompts:

1. Goal: identify the variables
 - Ask: what’s the primary action, i.e. what do they have to specify in order to enact the decision?
 - Use VCR; visualize, conceptualize (it’s a production decision, for instance), reframe (fill in the blanks, rewording as necessary: they have to decide how much of/many of each type of ___ to ___)
 - Amend R to account for 2 or more levels (e.g. how many pounds to ship from warehouses to factories)

2. Goal: identify the objective function and the constraints
 - Look for limitations, explicit or not, and requirements, verbal description only
 - Use the taxonomy: what processes do the variables go through, what affects their availability (supply) and demand, what interactions, e.g. percentages, are imposed on them
 - Note the units defining each constraint

3. Goal: identify numeric information
 - Separate totals from unit quantities in non-relational constraints
 - Align them with the appropriate equation or objective function
 - Note the specific percentages and other relational numbers

4. Goal: complete the equations
 - For (the objective function) and each constraint, ask which variables are relevant

- Focus on the units of the constraint; ask if they differ from the units of the variables
- If they do, use unit matching: to find or compute the coefficient of a particular variable, ask “how many of those (units of the constraint) do I need/get for each of these (units of the variable)?” Here is where the unit quantities enter in and should be accessed during this highly directed reading
- If they do match the units of the variable, unit matching may still apply; usually, however, reasoning and translation will work, illustration forthcoming

“The Consolidated Grain Company owns two farms, one in Iowa and one in Indiana. The Iowa farm is 300 acres, has available water of 250,000 gallons per season, and costs \$100 per planted acre per season to operate. The Indiana farm is 240 acres, has available water of 400,000 gallons per season, and costs \$200 per planted acre per season to operate. Consolidated plants three crops on these farms: wheat, corn, and barley. They estimate that wheat will sell for \$25/bushel, requires 1000 gallons of water per acre, and yields 20 bushels/acre. Corn will sell for \$20/acre, requires 1500 gallons of water per acre, and yields 15 bushels/acre. Barley will sell for \$15/bushel, requires 1800 gallons of water per acre, and yields 40 bushels/acre. The company forecasts a maximum demand for wheat of 5000 bushels; for corn, 6000 bushels; and for barley, 4000 bushels. However, at least 2000 bushels of wheat is necessary.” (Gordon & Pressman, 1983, p. 86).

On the first pass, one might be able to take the two sentences “Consolidated grain company operates two farms, one in Iowa and one in Indiana” and “they grow three crops, wheat, corn, and barley” to realize, with the help of the reframe, that they have to decide how to plant. Specifically, they have to decide how much of each crop TO PLANT on each farm; or they have to decide how much land TO SET ASIDE/ALLOCATE for the planting of each crop. All the details fade away. The second pass may lead into the second read, as one finds details, but ones that identify constraints, such the size of the farms, the amount of water, and the unit cost of operation (which helps identify the objective function). Continuing with the second pass of the second read, when one encounters the sentence “wheat yields 20 bushels per acre, sells for \$25/bushel, and requires 1000 gallons per acre per season,” revenues become a part of the objective function so that the focus is on maximization of profit. Water is seen as a limiting constraint.

That sentence sets up the third read because it is all about unit quantities just as the sentence about Iowa gives us the necessary totals. Using the matrix methodology and the meta vocabulary, students are likely to be less confused at

first recognizing these numerical bits of information. Then without actually computing anything, they can position the unit quantities by linking them to the relevant constraints and/or objective function.

In queuing theory, the multiple reads with differentiated goals can be presented as follows:

1. Identify the system
 - Ask: what's the service, using an "ing" ending for the verb describing it
 - Ask: who/what needs it/requests it; all possible such individuals
 - Ask: who/what provides it
 - Ask: how do customers wait for their service to begin – do they wait in line, in an electronic queue, or just sit there (e.g. machines)
 - Ask: does the population (all potential customers) have a cap?
 - If not, does the queue have a cap (ask if at any given time, there is a maximum number of customers allowed to wait)
 - If neither of those apply, model type is basic
2. Identify in meaning only the standard queuing variables and inputs
 - a. Ask: how often, on average, do customers request service (using the specific versions of those generic terms)
 - b. Ask: how long, on average, does it take to complete a single transaction
 - c. Draw a time line to indicate what it means specifically to enter the system, to have service begin, and have service end
3. Identify the problem type – compute an average v. probability; if probability, direct computation or criteria specified

Conclusion

Many skills college students apparently have in place are not taught, at least directly, because "they're supposed to know that/be able to do that" or because it simply doesn't occur to instructors that they don't. Moreover, instructors often project and thus assume that students' internal processing mirrors their own. Such is the case with reading as it pertains to complex verbal problem-solving. The argument presented here takes issue with that position, building a model for reading and then extracting certain scenarios as promoting poor performance. Focusing solely on the reading function in the context of problem-solving, the article then makes suggestions for how to improve performance, based on this model.