In teaching mathematical subjects, especially in the context of solving "word" problems, I ground my approach on the premise that poor student performance flows largely from self-imposed tacit restrictive assumptions – rather than from lack of innate ability or lack of effort. These assumptions drive reflexive, i.e. automatic behaviors that prove counterproductive. These assumptions exist on content, process, and self-concept levels; the latter two bear the brunt of greatest responsibility in resulting performance and experience. Content-based assumptions have minimal impact because they have less to do with the student's identity than process and self-concept assumptions; in fact, they do their primary damage by triggering preconceptions about what one can and cannot do (process) as well as what it means (for the self-concept) to encounter difficulties along the way. I find that these assumptions manifest most insidiously and powerfully in the act of *reading*. Before getting to that and offering ways out of self-imposed binds, I examine the assumptions.

**Process Assumptions**

I want to focus now on some typical preconceptions novice problem-solvers tend to hold when they perceive threat in the task of transforming given verbal and numerical information into a workable format that then leads to a useful conclusion. They tend to deny valuable resources, in particular time, process, and recourse. And they effect this denial tacitly, thus outside their awareness.

The following table shows the links between such assumptions and resulting behaviors:

| ASSUMPTION | BEHAVIOR |
|---|---|
| No time | Process immediately, especially details<br><br>Undifferentiated re-reading |
| No process | Wait for solution to emerge as a whole<br><br>Fail to set up insight with routines |
| No training (prototypic methodology<br><br>extends) | Rely tacitly on previous strategies<br><br>Narrow focus to the problem at hand |
| No recourse (information inviolate);<br><br>(can't get back on track once off) | Take notes, but don't take note<br><br>Don't look back; move forward only<br>Don't change the approach |

Most of these are self explanatory, but some elaboration should prove helpful. With the “no process” assumption, a concomitant behavior, failure to set up insights with routines, refers to the fact that an important aspect of process is the link between various immediately implementable actions and a subsequent discovery. With the “no training” assumptions, students act as if there are no problem-solving strategies to learn from the course itself; they can only extend what they know how to do in a non course specific manner (a naïve approach) or they try to extend the original example they learned. Unfortunately, neither assumption serves them over time, in fact acting as a concealed contributor to poor performance.

I now turn to the main behavioral culprit in creating poor performance, namely, immediate – and generally premature – processing of information. Students tend to try to move toward a solution both ASAP and ASAR (as soon as they receive it = the information). This leads to unrealistic expectations of self along with/because of lack of efficacy in (1) storing and (2) manipulating information.

Specifically, with respect to storage, students typically act as if they must hold the information in their heads ready for immediate use, details and all. This assumption flows from the “no process” and “no time” beliefs: threatened students in particular treat the information as of a single piece, as if incapable of “unpackaging” and then transforming it. They further proceed with the belief that they must get it all on one pass, i.e. they cannot reread but instead will only see the details one time. (If they do conceive of rereading, they tend to simply go over the same ground, hoping it will somehow magically click in – and assuming that for those who succeed, the magic is there, born out of their superior talent).

So when the problem presents some complexity, making these behaviors unworkable, such students tend to think themselves inferior, projecting their beliefs onto their more successful counterparts and also onto the instructor. In other words, they assume that those who continue to flourish in this more difficult problem-solving context do so with the same assumptions and strategies that may have worked on simple end of the chapter textbook exercises. It rarely occurs to these students that success flows from a different orientation and set of behaviors.

The issue then becomes one of managing student process and self preconceptions in real time as they interject themselves into the working of a problem. In particular, instructors may benefit by narrowing the focus to the act of reading itself, seeing this as a repository for the restrictive assumptions and subsequent ineffective actions taken.

*Reading: the quiet assassin*
Why would one take a look at how students read? After all, it would seem late in the game to effect major changes in that arena. Consider, however, that the act of reading constitutes their first encounter with the problem. After I advise them that assumptions about how to proceed typically manifest themselves in the general behaviors cited earlier, I then tell students about the seven deadly sins of reading:

1. Getting hung up on a detail
2. Trying to do math manipulations as soon as they see numbers
3. Engaging in "private reasoning" and ignoring the concepts of the course to figure things out – especially, through visualizing
4. Not trying to connect to previous problems; or trying for a whole connection to a previous problem (rather than several small connections to several other problems)
5. Failing to use prompts to help themselves bridge the gap between a concept and its specific form in the problem they're working
6. Assuming they must work on the information in the order received
7. Rereading without having interim goals

In this essay, I want to concentrate on how details, both verbal and numerical, sidetrack effective reading. I will use one main example in which students typically felt inundated by numerical and verbal information, yet felt compelled to do something with it – or else give up almost immediately. The way out of this bind requires students to refrain from immediate processing. The trick to implementing such a strategy centers around providing them with another direction to pursue; I try to persuade them to read *structurally*, illustration forthcoming. Structural thinking is defined here as: the identification of individual entities and relationships between them within the subject and then applied to the problem; the explicit connection between a course generated concept in the abstract and its specific version in the problem; the construction of a typology of scenarios designed to identify basic aspects of the problems students encounter; and finally, distinctions between conceptualizing, communicating, and computing as relevant activities of problem solving.

**<u>Examples</u>**

Larry Buckly, the director of the computer center at Edison College, is charged with maintaining cost effective staffing of the center while still meeting student needs and certain union requirements. The center is open from 8 AM to midnight. Larry has researched student usage of the facility and has been able to break down staffing needs for four distinct four hour time periods. Specifically, a minimum of 4 "consultants" should work from 8 AM to 12 PM, 8 from 12 to 4, 10 from 4 to 8 PM, and finally 6 from 8 to closing. To complicate matters, Larry has to use a mix of full-time consultants paid $14/hour who work any of three 8 hour shifts, the first from 8 to 4, the second from 12 to 8, and the third from 4 to midnight, and part-time consultants paid $12/hour, who work only four hours, beginning at 8, 12, 4, and 8. To protect the full timers, the union insists that for every part-time consultant on duty, there must be at least two full time consultants,

Whew! Anyone reading this without the benefit of a management science course is going to say, "No wonder they have a hard time." Students do have a conceptual framework called linear programming to use in their analysis. Nonetheless, most of them react the way most of you just did, feeling overwhelmed by complexity of detail and then reading/acting in counterproductive ways, cutting themselves off from the resources they need.

The details do not have to be quantitative. Sometimes, just because there are so many of them, paradoxically, many students feel they must address them ASAP – as indicated earlier. In this problem, the introduction of a director, with name and title, was not usually perceived as a detail to remember or manipulate. With just that individual's name and job description, most students used their judgment and recognized they could put that in storage without attempting to remember it -- they saw that some individual did the scheduling. But when I experimented and added further complexity by introducing 4 other staff people with different responsibilities, names and titles along with some indication of their activities as related to the problem at hand, some students grew uneasy, struggling with and getting hung up on the level of detail accompanying job descriptions, under the assumption that these distinctions were somehow (immediately) relevant.

Reading structurally requires several passes at the information, each with a different goal. First, students need to articulate what any reader who <u>hasn't</u> taken the course could say: a director has a staffing decision to make; 2 kinds of workers are involved; quotas exist for certain time periods; and there is a relationship between the numbers of the two kinds of workers. This mini analysis is well within their reach and does not rely on any course related concepts.

On the second pass, I suggest that they make as their goal the <u>connection</u> of the problem particulars to the conceptual framework applicable. Here the course begins to have an impact – but students still need to postpone processing any details. In linear programming, the concepts of note are decision variables (here, how many of each type of consultant to hire on each shift/time period), the objective (here, to minimize cost), and constraints (here, the requirements in the form of quotas and union regulations). I refer to this stage as "seeing as": each of the four quotas is recognized as a constraint, with numerical and even some verbal details not yet acknowledged.

To help them with this stage, I prompt them to pause at each information fragment and ask "is this up to them or determined by something else beyond their control?" to separate variables from constraints. Then, with a constraint, I encourage them simply to name it, e.g. "quota for 2d time period." It is also helpful to see how various aspects of the problem at hand compare with previous problems. For example, the quota constraints are structurally similar to advance orders for various products, in establishing a minimum demand. This cross comparison continues throughout the analysis. Stage 3 is next.

Here, they begin to translate, setting up their own symbols for the decision variables. This involves the necessity of making distinctions in code, while taking care to be explicit and precise. So saying they need to hire a certain number of full timers and using the letter F will not be useful because the company must specify how many full-timers will be hired when: something like F3 for full timers on the third shift, for example, would do. Still no numerical details, but hopefully the systematic approach combined with discipline are being established.

When they continue and begin to express relationships, they may bump up against some assumptions on a content level. In this problem, students may feel uneasy and confused by the simple fact that the shifts overlap: 8 to 4 and 12 to 8, for instance. Many automatically assume no overlap. Students will inevitably make such assumptions and do so without awareness – just assuming. On a psychological level, they need to confront this issue and not cop out by saying "I am not smart enough to handle this stuff."

Thus, I present at least 3 reads whose ultimate goals are conceptual and symbolic recognition with a minimum of detail. Overarching goals include adopting useful strategies, working systematically, and maintaining discipline. (They are also learning to read!). And for subsequent passes at the problem, they have now set up a framework in which the details can be smoothly assimilated. Mistakes will occur, but less frequently and with the underlying assumptions closer to the surface.

Linear programming ultimately calls for writing a system of equations that express the constraints and the objective in terms of the decision variables. For an example, consider the second quota, that at least 10 consultants be on board between noon and 4 PM. Instructors can offer a prompt by having students ask in general, which variables are relevant/involved; specifically in this problem, we ask, who's (still) there? The full-timers from both the 1$^{st}$ shift (their last 4 hours), the full-timers from the second shift (their first four hours), and the part-timers hired for that time period.  To express that in equation form, they must simply translate:

$$F1 + F2 + P2 >= 10$$

Another content assumption held tacitly is to also incorporate the relationship between full and part-timers into this equation.  Let's backtrack first and deal with the requirement that there must be at least two full-timers for every part-timer.  Simplifying our example by looking at the first time period with just F1 and P1, many students expressed that relationship as

$$2F1 >= P1$$

I inquired about this error (the correct equation is F1 >= 2P1) and was able to help them realize that they put the 2 next to the F1 from visualizing the F1 rank and file as twice the size of the part-time group.  I distinguished this tacit internal picture and copy approach from the explicit process of translating the requirement into code, i.e. symbols and numbers.  I also encouraged them to make up some numbers, arrive at a result and then express what they did to get that numerical result in general, i.e. symbolic, terms, to find out where the 2 should go.

But back to the quota; as indicated above, some students tried to include this equation within the first, i.e. to combine the two.  This proves to be unnecessarily restrictive.  It is necessary to separate the two requirements, deal with each individually in terms of setting up an equation, and then let the computer sort it all out.  The main point is that assumptions are everywhere!  All the while, I discuss the role of these assumptions in contributing to poor performance so that students get away from the lack of innate ability as the reason for it.

Summarizing the reads:

1. Ignore all details, focusing on the problem structurally out of discipline
2. Ignore all details, now seeing them as representative of discipline-specific concepts; compare to other problems
3. Sharpen the definitions of the specific versions of the concepts and translate them into symbolic code; focus on individual entities and reread for necessary information
4. Express relationships by translating; reread to incorporate details
5. Compute or use software to get some sort of solution (we left this out of our example); reread to pick up details where needed

One of the instructor's major roles during any of the reads and activities is to help students surface blocking assumptions about content (what this concept must mean), task (what I must or must not do), and self (what this progress means, for me).

<u>Some others</u>

In another problem, a company that makes three ready-mixed concretes sought the best mix for the coming months. Some of the details involved the sentences describing the relationship between the type of concrete and its composition, e.g. "mortar is composed of 10% sand, 80% cement, and 10% gravel." Three such sentences taken together presented to many students an overwhelming amount of information. They felt – tacitly – that they had to somehow sort out and remember all of it before proceeding further. They fell into the trap of seeing many details instead of seeing one type of detail repeated many times. A simple shift in perception – once we realize how students focus – will help them relax.

But the real key to effective reading on the first pass again lies in simply recognizing that, for example, the company making the mortar, industrial concrete, and residential concrete has 3 products, all various percentages of three ingredients. Further, on the second read, students benefit from realizing and explicitly articulating these percentages as absolute: while not focusing on the 10-80-10 per se, they are in a position to contrast this set up from other input-output problems that allow for variability in such relationships (e.g. two types of jersey bands can be made from any or all of three ores by smelting these ores in furnaces specific to the bands). (By the way, how did you as a reader react just now to the term "jersey band," made up as a specific stand-in for the more generic "product"?)

**Building bridges**

Getting students to relearn how to read would seem to be a monumental chore, but the seeds have already been sown. For instance, almost no one in the class got hung up in our main example in its original form on one piece of information – the director's name and title. In other words, the students have already demonstrated the capacity to see beyond detail and know how to "see as." They also have been able with more limited success to recognize concepts in context. They have the capacity to think structurally, in terms of individuals and relationships, concepts and examples, varying contexts, and so on. They need someone to keep them in touch with these internal processes, one who believes this meta-level to be as important as the discipline itself. This essay has attempted to begin the building process through the royal road of reading. Ideally, the instructor turns the student from an individual who, because of an ineffective reading approach, perceives threat in apparent complexity, becomes closed and defensive, holds tacit assumptions, and makes many errors, to someone whose reading plan reduces and manages complexity, who welcomes a challenge, looks to surface assumptions and is willing to investigate errors. Any progress along these lines represents a crucial second level gain, i.e. one that transcends the specific course.