# MATH ANXIETY REVISITED

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# PART ONE: The math anxiety phenomenon

***Math anxiety*** has been of explicit concern in the United States for over 30 years. It has been suggested that it is so prevalent that it affects educational policy (Walker & Karp, 2005) and that many teachers in elementary schools are so afflicted. Burns (1998) estimates that 2/3 of American adults find mathematics intimidating. As a society, this implies that in order to increase our math-science “people capital,” we need to train teachers in the lower grades how to overcome this condition in addition to preventing its proliferation in later schooling (so that prospective teachers are more confident in their mathematical abilities).

This article focuses on the causes and treatment of math anxiety for college age (and older) students taking required courses with a significant percentage of mathematical content. It examines in detail instructor – student dynamics in the present while also looking at each party’s individual contributions to this condition. Such an analysis signifies a departure from others in this area where in particular, instructor contribution is typically considered only as a past-based phenomenon – as in, “I’m anxious because my 4th grade teacher made me feel stupid.” Lost in the genuine attempt to rid the planet of math anxiety is the fact that the current instructor may not only be triggering feelings from the past, but also truly making life miserable for students in the present.

I will proceed as follows. First, I will briefly review some current and standard definitions and explanations for math anxiety. Then I present a model that identifies sources of anxiety, linking them to performance and attributions. After attending to the factors behind the anxiety, I endeavor to indicate how they can be reversed in the second half of the article.

### Some definitions and assumed causes of Math Anxiety

In the broadest sense, anxiety can be interpreted as a reaction to a situation perceived as threatening (Barnes, 1984). Mathematics is often considered as a difficult learning venue with the potential to trigger feelings of distress and even dread in those students who are at a loss to comprehend its concepts or to use it to solve problems. This response is also taken as a definition of math anxiety (Ashcraft & Faust, 1994). There is a definition of a math anxiety cycle (Mitchell, 1987): past trauma in learning within this domain leads to present apprehensions, replete with physical symptoms and strong negative affect, creating a scenario in which the student cannot absorb information or think logically. This culminates in poor performance, causing even more anxiety and initiating a cycle with continued poor performance.

Ashcraft & Kirk (1997) include preoccupation with worry and self-doubt as math learners so that students cannot focus on the material or assigned problems, as an operational definition. Helplessness, paralysis, and mental disorganization when dealing with mathematical problems are the defining characteristics for Tobias and Weissbrod (1980). The hallmark, according to McKee (2002), is a state of confusion, panic, memory loss, and mental paralysis. Most of the researchers agree on the physiological symptoms describing math anxiety: increase in heart rate, sweating, trembling, and nausea top the list. Math anxiety has also been defined as the “state of discomfort” students experience in responding to mathematical tasks that threaten their self-esteem (Cemen, 1987).

When students are asked to reflect on how they came to be math anxious, common categories include bad/insensitive teaching in the past, gaps in learning, and attitudes of others (Freiberg, 2005; Perry, 2004). Hadfield and Trujillo (1999) offer a similar set of attributions from their research: “Environmental” factors include previous difficult classroom interactions, pressure from parents both to succeed and not to (gender bias: parents who feel it’s not appropriate for females to excel in math sabotage any interest in the discipline), bad teachers in early schooling, and the perception of the discipline as nothing more than a collection of rigid rules. “Intellectual” factors include learning styles not being addressed, low self confidence in math ability, and finding/dismissing math as irrelevant. The final category, “personality traits,” include shyness, low self-esteem, and gender (the field considered until quite recently to be a male dominated one).

Spielberger (1972) sees anxiety as resulting from a cycle initiated by a stressor and a perception of threat. An interesting point of view comes from Norwood (1994) who focuses on traits such as difficulty in handling frustration and impulsivity (as in ADHD), combined with poor self-concept. Dossel (1993) identifies several factors, such as the apparent right – wrong answer typically thought to characterize mathematics. Tobias (1993) posits that word problems are the scourge associated with math anxiety in part because students are rarely taught strategies separate from content to deal with them. Montague (2002) refines this attribution with the suggestion that anxiety flows from not knowing how to begin working on problems. Zopp (I999) points out that the anxiety in a learning context can result from unrelated life events, such as a death in the family.

Finally, school teachers are often math anxious themselves and inadvertently pass their feelings on to the more vulnerable students, as their lack of confidence in learning math is contagious (Fiore, 1999). It should not be surprising to find that parents can pass on their anxieties as well (Jackson & Leffingwell, 1999). Buhman and Young (1992) implicate the math anxious teacher in the lower grades, particularly those who cover up their deficits with a false show of confidence. This scenario can create a disastrous experience for some children, as I shall discuss later when I present my model.

*Another approach*

In reviewing these theories/frameworks, I find them to be essentially “static”: that is, they take math anxiety as something afflicting a student because of past events and relationships. While basically agreeing that these past-based factors are indeed relevant and in fact may fully account for a student’s state of mind entering a required technical college course, I am more interested in how what happens in the course can create, perpetuate, and exacerbate anxiety on the one hand or ameliorate it on the other. The dynamics between students and instructor, between students and the discipline are ongoing and I focus on them in the current course.

Although many articles also include recipes for amelioration of the anxiety, they act as if the current instructor has only to act on these suggestions, not taking into account that this instructor may be <u>adding</u> to the problem during the course for a variety of reasons. The tacit assumption in many of these perspectives is that college instructors do not themselves suffer from math anxiety and are therefore above scrutiny; they have only to be trained, to learn what to do. Thus, according to this way of thinking, students’ math anxiety will either be addressed in a helpful way if the instructor chooses to follow these guidelines or will remain as is.

I suggest that the students' ongoing interactions with instructor and discipline shape their responses profoundly. I shall construct a longitudinal model, beginning with anticipation of the course and concluding with performance, attributions for poor performance, and psychological consequences for students as a result of those attributions. I further suggest that amelioration can only be robust if it follows a detailed analysis of how instructors can create, perpetuate, and escalate the concerns of "pre anxious" and anxious students.

**The CCPP Model**

The model I propose is based on the central theme of anxiety as resulting from poor performance coupled with crucial concealed correctable contributors to that performance. In this model, labeled CCPP, four sources of (concealed) contributors are identified, with a flow chart that indicates the path from source to contributor to concealment to performance to attributions made by both the instructor and students to the anxiety, should it occur. The contributors are roughly in two camps: lack of access to critical resources and mismanagement of process. Typically, anxious students are unaware of such resources (as opposed to those who do not care to make the necessary effort to engage them) and have no sense of the way process has been mishandled. A second layer of concealment is the concealment itself. In other words, math anxious students do not know what they're missing and further, don't know that there is anything missing.

Not all contributors to poor performance are concealed for all students. For example, students may know they didn't make more than a token effort or may feel confident that their instructor did not adequately assist them. When the reasons for the failure are properly understood, students will not generally suffer from anxiety, at least not in its full flower. They are more likely to be math averse, often either because they lack the motivation to learn the subject and/or because of negative experiences in the past. Accordingly, I want to distinguish between 4 levels of (vulnerability to) anxiety:

1. The "pre anxious" student, one who has not had an anxiety reaction before, but who is very vulnerable because of a deceptively limited math background. This cohort consists of several subgroups: for instance, there are students who managed to avoid math in the past because it did not appeal to them (perhaps they attended schools that allowed students to fashion their own curricula). Then there are those who have succeeded and even enjoyed math in courses that moved along slowly and did not demand application to word problems; or those who succeeded when they took elementary required courses in high school, because they could get by with a healthy dose of memorization. These are students with a minimum of basic scholarly skills and untrained in mathematical/logical thinking in spite of their success. There are also cultural issues to consider: Treisman (1992) worked with freshmen at UC Berkeley who had high levels of achievement in very under demanding minority secondary schools, where success in math courses consisted of manipulation skills. Taking calculus, these students were often shocked by the rigor demanded, the pace of the course, and the focus on problem-solving. Until a special project was put in place to assist them, most fell by the wayside with D's and F's.

2. The aforementioned math averse student, who has typically had enough negative experiences in attempting to learn the subject that s/he is apprehensive about any

future endeavors. This student anticipates a new course with some worry (cognitively), is tense and edgy (physically), is somewhat fearful (emotionally) and tends to jump at new concepts and rush to solve problems without taking sufficient time to fully understand them (behaviorally). Yet this student still has some hope s/he will succeed, at least to some extent, believing that prior performances could have been improved either because s/he knows that his/her effort was minimal and /or can fault previous instructors for poor explanations and uninspired lecturing. In some cases, math averse students have seen through one layer of concealment: they know something is missing, but don't know what. In other cases, they also see what's missing, but don't make the effort to take it in.

3. The math anxious student (fitting many of the standard definitions). These students have felt lost and confused in prior courses. They have often put forth a solid effort into previous courses and still not gotten it. These students anticipate a new course with great trepidation (cognitively), sweating and trembling (physically), feelings of dread (emotionally), and automatic responses that include mental paralysis (behaviorally). They have little hope of succeeding in the course. Further, should their performance be expectedly poor, math anxious students rarely blame the instructor for their plight, at least in the present, considering him/her to be a messenger for the purity of math, which is never wrong and therefore never to blame. Instead, they typically attribute failure to talent, i.e. to the lack of inherent ability, although they sometimes realize that their anxiety can also play a major role in the outcome.

4. Finally, the super anxious student has internalized concealed anxiety from an <u>instructor</u> following a poor performance and a joint attribution that the student's lack of inherent ability is the cause of the failure. In effect, the super anxious student colludes with the instructor who needs to believe that everything possible has been done. This response stems from a universal interpersonal dynamic in leader-follower contexts in general and instructor-student in particular: an instructor who by disguising his or her own charged affect from students, even when not intending to do so, can "transfer" that affect to them (Alderfer, 1993). Thus, when instructors blame students for poor performance in however gentle and subtle a manner while simultaneously able to mask any of their own fears and concerns with an appearance of self-confidence, they amplify the student's anxiety to an even higher level. The mechanism seems to be that students inadvertently "pick up" an extra charge of these fears, adding to their own related feelings, but indistinguishable from them. So, on top of their anxiety from a poor performance with concealed contributors, there is now an additional level of concealment. The student cannot find an external locus to account for this extra heavy burden of self-doubt. This is a particularly pernicious form of anxiety leading to a condition known as "learned helplessness" (Seligman, 1992), characterized by personalizing the failure and assuming it to be pervasive and permanent.

The standard attributions in any learning endeavor (Martinko & Gardner, 1987) for sub par results are R.A.T.E.D., meaning that poor performance is due either to <u>r</u>andom factors, <u>a</u>nxiety, (lack of) <u>t</u>alent, (lack of) <u>e</u>ffort, and the difficulty of the <u>d</u>iscipline. These attributions are not typically helpful as they do not pinpoint causes in an actionable manner; with the exception of effort, there is no recourse for any of them. The CCPP model, on the other hand, focuses on a set of more specific concealed yet **correctable** contributors – rather than the standard R.A.T.E.D. attributions.

*Shifting sands: change of student status from the current course*
I now consider the effect of poor performance in the current course on the three at risk cohorts in terms of change of status (students who are already super anxious are highly unlikely to take any new course that features mathematics): in other words, does it perpetuate or escalate students' anxiety profiles?

Students in the first (untrained) cohort will likely become math averse but not anxious if they suffer a poor outcome in the course. Should their instructor consider explanations other than lack of talent, that is the expected outcome; thus it means an escalation to the next level. Only if the instructor is very persuasive in stressing talent as the key while concealing his or her feelings and worries about contributing to the poor performance, is there any chance of escalating two levels to the math anxious state. It would indeed take extraordinary circumstances for an escalation to three levels, i.e. to super anxiety, but that cannot be entirely ruled out.

The math averse student who performs poorly will likely remain in the same position, status perpetuated, unless the instructor is able to convince him or her that lack of talent is the most crucial determinant of the outcome. Only then will student's status change and escalation to math anxious be the typical result. If the instructor is very persuasive while concealing his or her feelings and worries about having contributed to the poor performance, there is a real chance of a two level escalation to super anxiety.

For students already math anxious, an instructor who attributes poor performance to lack of talent and in the process, conceals his or her concerns (about the help or lack thereof s/he gave these students) is likely to bump the anxiety up to super anxiety. On the other hand, if the instructor is open in his/her judgment, student status is perpetuated rather than escalated.

I rely on two frameworks, one from cognitive psychology referred to as the ABCs (Arnkoff, 1980; Bedrosian & Beck, 1980; Hornby, 1990) and the other the concept of emotional intelligence (Goleman, 1996), to ground the model.

*The ABCs*
One stream of inquiry that applies to the math anxiety conundrum comes from the field of cognitive psychology. It would have us focus on an individual's responses to encounters with new concepts and word problems in terms of his or her emotions, assumptions, and subsequent behaviors. Its analytical unit is the ABCs – affect, behavior, and cognition surrounding an event. As a system, these three affect one another, with the typical sequences actually CAB or ACB, perception considered as a precursor. That is, once an event is perceived a certain way, assumptions (cognition) or emotions (affect) are activated (triggered). Either cognition drives affect, generating action; or triggered emotions generate action via resulting assumptions (Bedrosian & Beck, 1980; Hornby, 1990).

An individual's ABCs are considered to be <u>open</u> if a situation perceived as potentially threatening is reconfigured as a challenge and an opportunity; the individual is in touch with his or her affect, and that affect is generally positive; the individual's assumptions about the situation tend to be realistic, articulated, testable, and expansive; and the individual makes strategic behavioral choices in the service of growth rather than the avoidance of discomfort.

On the other hand, ABCs are considered closed if the individual is basically out of touch with the feelings generated by the threatening encounter, experiencing them as states of discomfort such as frustration rather than as specific needs and fears; the individual's assumptions about the meaning of the situation tends to be unrealistic, tacit, and restrictive; and the individual's behavior is constrained, automatic, and in the service of avoiding discomfort (relief-seeking). Obviously, these descriptions represent two poles of a continuum, with individuals falling somewhere in between.

I will focus throughout the article on the CB portion, since in a learning context, closed assumptions typically drive the automatic behaviors. In addition, there are important scenarios with commonly held assumptions and behaviors, while affect is often idiosyncratic. This frame will be applied to students.

*Emotional Intelligence*
Emotional intelligence (Goleman, 1996) tends to be used as parallel to "intellectual" or scholarly intelligence. This concept/framework will be applied to instructors. To have a high EQ (emotional IQ), an individual must be relatively self-aware and self-monitoring in the face of internal as well as interpersonal pressures. In addition, s/he is typically empathic and caring. Low EQ, on the other hand, manifests itself in "acting out" of these pressures by protecting oneself from them through a variety of defensive behaviors, such as being indifferent to others and refusing to take responsibility for his/her feelings and actions (Goleman, 1996). For example, the person who blames others and the "system" for anything that goes wrong, who does not take into account his or her own issues would be acting out. This instructor may be unaware of his or her feelings, or of where they are coming from ("of course, I get mad; I hate it when students are that stupid!"), and may feel no reason to regulate his/her expression, e.g. sarcasm, frustration, etc. Or an instructor who needs students to be like junior colleagues, but is out of touch with the driving force, may inadvertently expect too much; or, on the other hand, an instructor who needs to be seen as a guru may act out by intimidating students with his/her knowledge and acuity. EI is generally developed or enhanced through training although there are individuals who have a high EQ without it.

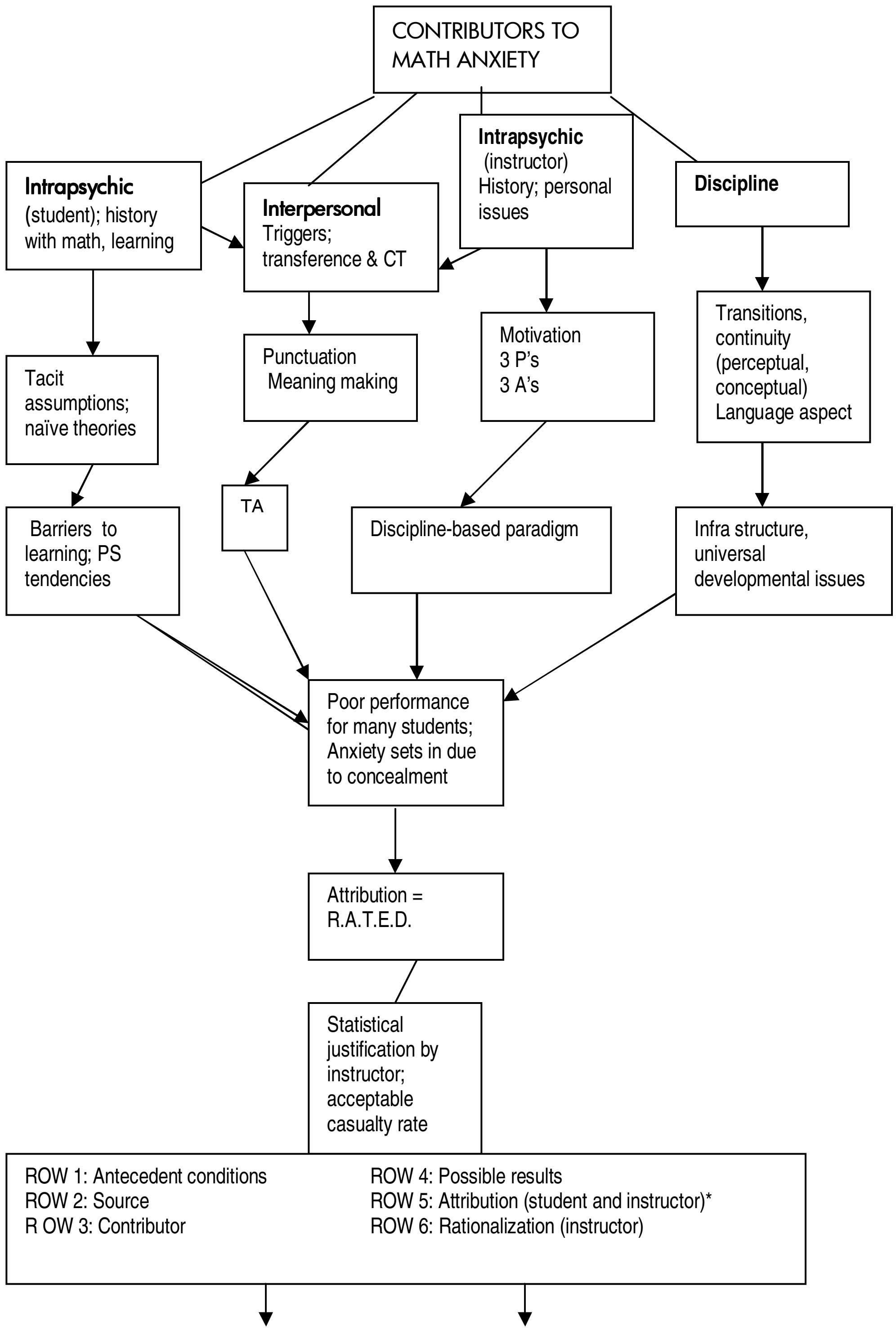
Emotionally intelligent instructors are not only are empathic, caring, self-aware, and self-monitoring, but also in touch with interpersonal processes such as projection and acting out. They are positively inclined to explore others' feelings and perspectives.

I contend that those instructors who believe that such training is beyond their purview or simply not something they wish to pursue are contributing to students' learning issues and may be rationalizing the threat of dealing with negative student affect. They are acting out by protecting themselves, demonstrating low EI in the process.

ABC analysis can be applied to both students and instructors, although I will use it primarily for students. EI also can be applied to both, but it will typically be considered as an instructor issue as it affects student performance.

Now consider the model schematically presented in Figure one.  Four distinct sources of poor performance are identified.  I will take each in turn and expand on it.  There are many terms in the diagram that have yet to be defined; I will clarify as I work through the model.

Figure 1




SELF-SEALING LOOP; SELF-FULFILLING PROPHECIES

**Student internal processes**

From this figure, the causes and dynamics of math anxiety can be verbalized. The student's cell at the second row left indicates that s/he creates tacit, closed assumptions for dealing with the concepts and tasks of mathematics. Part of the impetus for so doing lies in the student's previous encounters with the discipline, represented in the top left cell. He or she bases these assumptions on private untested theories about what seemed to work and what seemed not to. Some of them are meant for self-protection, such as "if I can't get it right away, I'll never get it; so I won't make more than a token effort after my initial attempt." Some of them have to do with learning in general, or more pointedly in venues for which rote memorization is not a viable approach: There is something to "get" and that may itself be threatening. Additional student problems arise from lack of basic scholarly skills, beginning with reading and organizing information.

*Problem solving*

Students who perceive the informational complexity of word problems as threatening often do so because they find the level of detail quite daunting. Starting with the first at-risk group, the untrained students, a common set of tacit assumptions consists of the following: "no time," "no training," and "no process"; (add "no recourse" for averse/anxious students) (Schoenfeld, 1989; Sankowsky, 2001). As a result, these students typically develop automatic and generally counter productive behaviors around problem-solving, reflecting a closed ABC stance, in contrast to experts'. Figure 2 shows 11 common differences between novices and experts, or between a closed, untrained stance and on open trained one.

Figure 2

| **NOVICE TENDENCIES** | **EXPERT TENDENCIES** |
|---|---|
| *Narrowing focus to the problem at hand: looking for a macro link* | *Widening focus: looking for multiple micro links* |
| *Engaging in "high level" reasoning*<br>(far from raw data, starting at most advanced point possible, taking others' interpretations to build on; not questioning the source) | *Keeping to fundamentals*<br>(reasoning within a simple framework from most elementary point possible; in particular making their own inferences from the data itself) |
| *Immediately processing information* (doing anything, jumping to conclusions ASAP, while also trying to hold details in one's head) | *Noting & storing information*<br>(establishing a process by first just categorizing, setting up for subsequent manipulating of information) |
| *Acting mechanically; not connecting to a framework or ignoring a framework*<br>**(**working without a framework, following a format without meaning) | *Connecting to conceptual frames*<br>(making sure that a framework fits the situation) |
| *Preserving order of information*<br>**(**working on the problem consistent with its presentation**)** | *Maneuvering information as needed*<br>(Order of presentation can be transformed to meet the needs of the problem solver) |
| *Moving forward*<br>(act as if current presentation inviolate and complete; stopping and struggling when stuck) | *Using self as barometer*<br>(knowing when to go back and clarify information; sensing when more is needed to make sense) |

| | |
|---|---|
| *Relying on action alone in problem solving* (focusing solely on what to do and thinking of available resources in those terms) | *Incorporating prompts and queries* (seeing action as flowing from reflections and various cues (prompts), and developing a repertoire) |
| *Doing one problem/project at a time* (proceeding vertically, from start to finish on one project/problem) | *Doing many projects/problems at once* (seeing problems as a group, with "cross-pollination"; proceeding horizontally, with one aspect from problem to problem) |
| *Rereading without interim goals* (going over the same ground several times, but without direction) | *Establishing interim goals for several reads* (rereading several times but with a distinct goal for each pass) |
| *Relying solely on visualizing in comprehending concepts and equations* | *Having a repertoire of situational responses for comprehending concepts and equations* |
| *Relying on verbal cues as a shortcut* | *Relying on concepts* |

First, by narrowing focus, students inadvertently under-emphasize any connection to previous problems, thus acting without the guide of experience and cutting themselves off from conceptual models (Santhanam & Sein, 1994). When they do see a connection, it is generally *macro*– "this problem is just like that one" – rather than *micro* and *multiple* – "this problem is like problem A in one regard and like problem B in another," the approach favored by experts. Further, students tend to lack structure-content balance: they get "inside" the problem or concept, focusing on the details and the specific defining characteristics, but do not make the transition to seeing it as a type of concept or problem.

Second, reasoning at a high level of inference has been shown to be ineffective in dealing with a variety of problems (Argyris, 1990). When students in any discipline tackle a difficult case, they will benefit from getting as close as possible to the raw data. The fifth and sixth tendencies speak to this issue as well, stressing the need to fully understand the problem before attempting to solve it by asking questions for clarification.

Third, by immediately plunging into mathematical manipulation, students effectively ignore the interaction between qualitative and quantitative information, thus bypassing the establishment of fundamental underlying frameworks. Students have a tendency to focus on details rather than structural and general concerns (Rentsch, Heffner, & Duffy, 1994). Their chances for committing errors increase, but they are frequently pushed by panic to rush to equations and other technical expressions. That this approach does work when dealing with one-step exercises confuses individuals assuming continuity: i.e. students who have been successful in previous "cookbook" math classes where facility in mathematical manipulation constitutes the primary skill required are often unprepared for and surprised by the process demanding complex verbal problems.

To illustrate, consider the plight of one student approaching a complex problem who tried to work on solving it as he read, a premature venture into solution building. He got in trouble because he assumed that the first company mentioned in the problem was the one "of record," i.e. that the problem sought to describe its perspective, in terms of decisions, uncertainties, and monetary outcomes. So when he read the second sentence and understood that it had to choose a location for a new facility, he correctly identified that as its decision, but erred when he dealt with it as such. For it turned out that the problem's concern lay with the third company involved, the one buying lots for new homes and hoping to build near the new location, having to decide where to buy land before the first company chose the location. Thus, the first company's decision was the third company's uncertainty and had to be treated as such in the analysis. I intervened in his process once he was thoroughly confused, having him reread the problem while holding him to first identifying the company of record as his only goal. When he realized that the third company had the difficult decision to make, he was able to correct the error and proceed.

Fourth, acting without a framework is a regressive tendency, generally occurring when a student feels so overwhelmed with the detail that that s/he perceives the framework as a burden in itself, simply adding to the problem rather than providing a direction.

For example, the framework needed for decision tree analysis can be presented here in a brief version because of its simplicity. On a qualitative level, it takes a specific well-defined business scenario from one organization's perspective. The issues it deals with are the company's decisions, spelling out the choices perceived by the organization as actionable, the uncertainties it faces, spelling out the possibilities it perceives, and the modeling, spelling out the assumptions the company makes and the limitations it perceives, both as absolutes. The quantitative aspect consists of monetary outcomes and probabilities with some simple arithmetical rules for combining them to come up with averages. The tree is a chronological flow chart of all sequences of decision/choices and uncertainties/possibilities. A branch on the tree is a particular sequence of choices and possibilities associated with the accompanying decisions and uncertainties, ending when a monetary outcome can be specified. Anxious students typically are so overloaded with worry and fear that they cannot absorb this basic framework, even though most of its terminology and concepts are well known to them, albeit in a more informal way.

Fifth, by processing information in the order presented, students assume a passive posture and quickly become overwhelmed by the details piling up in front of them. They act as if they must understand all these details and hold them in their heads. For example, when encountering a problem (using decision tree analysis) that introduced the company of record with the phrase, "JM, a small manufacturer of metal parts, was debating whether or not to enter the competition to build a transmission housing for PROTRAC," many students stalled because they felt that they had to understand what a transmission housing was before proceeding. All they had to note in starting their analysis was that the company had a decision to make.

Seventh, by relying on action alone, I mean that many students are unaware that they would benefit from cues and prompts to help themselves over the "hump" of a difficult word problem (Sankowsky, 2003).

Eighth, it seems only naturally to complete one problem before proceeding to the next. This is considered "working vertically." The alternative, one that emphasizes connections to other problems, is to work horizontally, setting up the same aspect/stage of several problems with no reason to assume any content similarities. Lowenstein, Thompson, and Gentner (2003) experimented with this issue by having two groups of MBA students compete against each other in an exercise involving 2 separate case analyses . The first group had a vertical approach, doing one case after the other, while the second group proceeded horizontally, looking at the same aspects of both cases simultaneously. The latter group put forth a much more solid analysis.

Ninth, by simply rereading the information, whether describing a concept or presenting a problem, most students in effect proceed with an undifferentiated and non strategic approach. They miss out by not having different goals for each read through, a strategy experts embrace, as will be discussed in the next section (Scardamalia & Bereiter, 1991).

Tenth, it may seem counter intuitive to argue that one should not rely on visualizing in a math course. It would seem at first glance that one needs to do more of it, not less. In general, visualizing is considered a positive aspect of managing one's activities. For instance in many sports, athletes are encouraged to picture themselves performing at a higher level in order to raise their performance. Cancer patients have sometimes used a visualization technique that has them making a mental image of a shrinking tumor.

The problem in this learning context is with visualization as a one-dimensional way of encountering data. It often manifests itself as ineffective and naïve "private reasoning," i.e. the eschewing of the canons of public discourse – marked particularly by untested assumptions and attributions. As a result, students are susceptible to poor performance (as a consequence of having an inadequate response repertoire, whether limited to visualization or other one dimensional approaches).

The overemphasis on verbal cues represents another dysfunctional attempt to reduce complexity. In tree analysis, for example, students often mix up decisions and uncertainties. Consider the sentence "The company may do X or they may do Y." In a hurry and pressured, some students jump on the verbal cue "may" as indicative of an uncertainty the company faces, as opposed to realizing that it has a decision it must make.

Failure to separate two pieces of information provided in the same sentence and thus verbally linked yields another example. For instance, a problem's first sentence read "the firm has two related lawsuits it can either settle or take to trial." As a result of the verbal presentation, many students saw one decision point with two choices: settle or trial. They did not see that the company had to decide how to handle each suit separately, possibly using the result of the first suit to determine its approach to the second. If they had waited until they read the second sentence, which explained that the suits will run consecutively, not concurrently, they would have been more likely to grasp that the firm had two decisions to deal with, with the resolution of the outcome of the first case having an impact on the other.

On the other hand, students frequently transgress on the side of redundancy. For example, a sentence in a problem outlining what would happen to a company if it lost a contract followed a sentence detailing the consequences of winning. Some individuals listed "winning" and "losing" as two separate uncertainties rather than seeing them as two possibilities associated with a single uncertainty. Similarly, using different verbal designations for the same idea often leads to the perception of two separate issues.

This characterization of novices' problem solving tendencies applies to a significant percentage of students, especially those in the first cohort: their lack of training pushes them to seemingly intuitive but counter productive behaviors. Math averse and math anxious students also exhibit these behaviors in a very entrenched manner that prevents them from entertaining other approaches.

*Reading problems*

In conjunction with and in addition to these counter productive problem solving tendencies, poor reading strategies (Scardamalia & Bereiter, 1991) also play a major role. Students are generally untrained and unskilled in reflective reading and base their actions on some seemingly intuitive principles. They often feel overwhelmed by details, not knowing how to organize them and how to prioritize dealing with them. Many students exhibit the following automatic reactions under such duress, all emanating from closed assumptions:

1. Getting hung up on a detail to the point of being stuck – e.g. when they don't know what a word or phrase means
2. Trying to do math manipulations as soon as they see numbers
3. Trying to work on details without looking at basic structures
4. Failing to reflect while reading; just taking notes, but without taking <u>note</u>
5. Rereading without having differentiated goals

Consider the example about the company named JM and its bidding dilemma. Some students tacitly assumed, as indicated earlier, that they had to know what a transmission housing is before proceeding further. That's getting hung up on a detail.

The details do not have to be quantitative, as this example shows. Sometimes, just because there are so many of them, students often feel they must address them ASAP. On the other hand, they can become paralyzed if they do not understand the details as they encounter them. Students who come into the course already anxious are particularly susceptible to becoming overwhelmed and then to cease their efforts. This tendency is consistent with the third problem solving tendency.

Many students have a split perception of problems – as either easy or (too) hard (non-routine). Their finding complexity intimidating stems from the assumption that there is no process -- reflected in this split view. Specifically, they typically fail to connect routine maneuvers with insight. Put another way, they fail to see the relationship between the implementable actions they can take and the discovery, i.e. solution they must make (Catrambone, 1980). Moreover, they do not set up and rarely take advantage of a repertoire of useful prompts (Sankowsky, 1998).

What instructors see is the ensuing behavior, which is effectively "tri-polar": either students become intellectually paralyzed, doing nothing at all (the anxious group: blank sheet of paper syndrome); or they read the problem over and over again the same way each time, hoping it will come together and eventually make sense (the untrained cohort); or they immediately launch into action mode, trying to do something with the information as soon as they read, e.g. write an equation or simply take notes without changing or transforming what's presented (the math averse cohort). In all cases, they assume they must attend to the information in the order it is presented – perhaps a reasonable assumption for fiction, i.e. a story, but manifestly ineffective in this context (Sankowsky, 2003).

In the first scenario, students report that they can't seem to get started. The level of detail stops them. They are caught in a bind, between assuming they must deal with it right away and not having a useful strategy to move forward. If they are intimidated by numerical information, they seem to focus on it first; without a supporting qualitative backdrop, they simply don't know what to do. They perceive a sea of numbers and become effectively paralyzed as it all seems too complicated and confusing. When the instructor solves the problem, many wonder why they didn't see what to do on their own.

In the second scenario, students proceed by (1) perusing the information in the order presented and without priority; (2) accumulating and holding the information in their heads as long as possible; (3) recognizing when unable to absorb any further information and rereading; (4) in the latter case, rereading in an undifferentiated manner, repeating steps 1-3 with the expectation of going farther each time; and (5) consequently hoping that a solution direction will emerge after a few iterations. The problem with this approach is that it does not reduce the perceived complexity – it just helps the student become more familiar with it. In some instances, the hoped for breakthrough may occur, as familiarity is at times the spark that sheds light on details. Generally, however, it takes a more direct assault (Sankowsky, 2003).

Finally, some students literally jump at the details, making assumptions about what they mean and what they must do about them, particularly the quantitative side. They feel that they must get right to it, to the meat of the problem. They then tend to engage in private reasoning in a kind of free association process. Their ultimate products reflects this (lack of) process. In the second half of the article, I will propose a systematic approach to reading.

*Understanding concepts*
Students tend to tacitly assume that whatever has worked for them in the past should continue to work for them in terms of grasping concepts, or else conclude that they've reached the limit of their ability. This gets them in trouble when they need to make a transition to keep up their understanding. When they become lost or confused as they attempt the same approach that has been previously fruitful, they further assume that others are still approaching it that way, and so are just more adept. This undercuts their self-esteem as learners. Not knowing or not being advised to make such a shift in focus effectively denies them the resources they need to progress.

To make this discussion more concrete, consider the probability "law" of complements:

$$P(A') = 1 - P(A)$$

This states that the probability of an event not happening is 1 (100%) minus the probability of the event happening. For example, if the probability of snow is 40%, then the probability of no snow is 60%. Most students in an elementary statistics class feel comfortable with this because they can verbalize, visualize, find an example, and compute with relative ease.

On the other hand, when the formulae become more complex, students attempting to visualize and verbalize with the expectation of immediate understanding are sorely disappointed because that is no longer possible. Math anxious students had already assumed that they would inevitably come to that point. Math averse students tend to assume it's just gotten too hard for them; they had hoped it wouldn't come to that. Untrained students are just bewildered. The concealed contributor to failing to grasp these relationships is trying to keep a one-step visual/verbal frame. The confused students don't realize they're looking for understanding in all the wrong places – and that even mathematicians aren't visualizing per se, but rather have a new way of comprehending the formula, focusing on relationships and patterns instead.

Resnick (1983) notes that students encountering new concepts in math and science typically form "naïve theories" to explicate them. This may involve a prototypical example and extending and paralleling prior ideas. Naïve theories have the same relationship to formal definitions as private landmarks have to a street map.

When taken as the first draft of understanding, initiating a process that involves feedback, making multiple connections with existing concepts, and trying out various applications, naïve theories are productive. But when they are prematurely formed and then entrenched/closed, they lower the chances of students' "getting it." Students predisposed to anxiety tend to do just that, to jump at the first sense of making sense, and then hold on to that fledgling construct, building on it without feedback.

For example, the concept of multiplying by fractions can be confusing if students retain the notion (from integral products) that multiplying means making more of something (Tobias, 1993). 0.36 times 8 does not carry over the sense of 36 times 8, in which students can envision 36 groups of 8 items apiece. 0.36 can be seen as 36/100, so that multiplication and division are expressed in one symbol; or the notion of a product with fractions can be rephrased as "of" rather than times.

In general, premature and entrenched naïve theorizing is based on the <u>continuity</u> assumption: that is, the concept should be understood in the same way as a previous one. More effective processing of such ideas involves finding a point of departure for them, seeing both the connection to the familiar and the distinction from it at the same time.

**Instructor contributions**

The focus on past bad teaching often obscures the fact that in the present courses, instructors' internal issues can also wreak havoc on student learning. One particularly insidious and concealed vehicle for helping them deal with their own issues at students' expense is a seemingly innocuous, benign, and even appropriate approach to teaching in general: a content-only stance, labeled here the discipline-based paradigm (Sankowsky, 1998). However, this paradigm can contribute to student poor performance by failing to take into account process and structural issues, particularly for at-risk students.

*The Discipline-Based Paradigm*

The discipline-based paradigm of instruction is grounded in the vision of the field of study as inherently valuable, with an accompanying mission to focus solely on <u>content</u> according to its internal logic and its applicability. This paradigm defines the instructor's role as provider of knowledge, based on his or her expertise, and students' objective the mastery of that content. *Process issues*, i.e. how students go about learning, teaching methods, the relationship between students, instructors and the discipline, and instructor-student dynamics, enter only tacitly into the equation: the discipline's rigor and lack of ambiguity ostensibly point the way unequivocally toward implementing an effective pedagogy, requiring clarity of presentation, patience with student learning efforts, and acceptance of a certain level of poor student performance as inevitable by the instructor. Typically, none of these issues are openly discussed.

(And they don't need to be discussed in higher level courses. It is particularly in the required intro technical ones, in which students are often presumed unable to entertain such analyses, that these aspects are called for.)

Focusing first on those instructors who look forward to teaching the required introductory courses, it is suggested here that (especially) in quantitative methods, this is the default paradigm on at least three levels: personal, psychodynamic, and political/professional. "Personal" refers to the instructor's passion for his or her chosen field; "psychodynamic" refers to the instructor's needs, fears, and other feelings possibly driving him or her in the teaching process and based on the interpersonal context; and "political/professional" refers to the instructional implications of the profession's needs.

On the personal level, instructors who love the discipline will likely gravitate toward the discipline-based paradigm. They find the discipline so compelling that they need to talk about its content as much as possible, with very little concern for the process of learning it. They want to share their knowledge with others, in particular treating students as junior colleagues.

On the psychodynamic level, instructors (like anyone else) have a tendency to generate assumptions about what to expect in the classroom. They may safely assume, for example, a high level of anxiety and a low level of interest in much of their audience, especially with required courses. In response to this anticipated and often realized reception, they find the discipline-based paradigm comforting, the focus on content a refuge from any interactive tensions.

On the political/professional level, instructors serve as advocates for the discipline. In this context, they seek to protect the profession's interests by competing (with other disciplines) for new "recruits." This impacts presentation both in terms of method (topics, focus, and pacing) and audience (the profession may favor an elitist orientation to grow its membership). The discipline-based paradigm satisfies this need efficiently and straightforwardly.

In general, these well defended instructors often don't get why the students don't get it; but students have a limited repertoire of resources for getting it when left to their own devices. And that's precisely what they're left to as those instructors shy away from any discussion of learning or feeling processes; they go right for the jugular, in this case, the content. For they believe that to be unambiguous and pure and elegant. They get that. That's why they are drawn to math in the first place. Typically, they didn't need any sort of process analysis.

Students often share in and extend this belief system: they don't want to talk about process either because they won't be graded on it and don't appreciate its relevance. The apprehensive learners just prefer to narrow their focus and "get it over with" ASAP, as if taking medicine. As indicated, they believe that if they don't get it right away or within some self-imposed time frame, they aren't ever going to get it. They then assume that they're just not cut out for it, that they have reached an inexorable limit.

There is also a set of rationales for choosing this paradigm when instructors are themselves required to teach the course, e.g. for departmental rotation purposes. Those are (1) affect, (2) agendas, and (3) anxiety.

By affect, I refer to the feelings underlying instructors' choosing to teach as a profession. There may be fears and needs that instructors would rather keep private, naturally enough, and so the discipline-based paradigm keeps the focus on the subject matter in an impersonal way. For example, s/he may need to have an audience marvel at his/her expertise and knowledge. On the other hand, s/he may feel judged by a large class in a required subject.

By agendas, I refer to internal politics (concealed from students) relating to teaching assignments, such as rotation, department chair favoritism, etc. For instance, an instructor may feel singled out by his/her chair to teach this elementary course if it denies him/her the opportunity to offer a seminar in his/her research area. The default paradigm fits here, because for instructors with this agenda, it requires the least expenditure of time and effort to prepare for it.

By anxiety, I refer to instructors who are apprehensive about either teaching in general or the course in particular. They may have entered the college teaching ranks in order to do their research. With no courses in education under their belt nor any real interest in student learning issues, they will find the discipline-based paradigm the most efficient way to quell their anxiety. The discipline is unambiguous and so they believe (by extension), that teaching it should be also.

In all these cases, the instructor may be concealing his or her true motivation for choosing what appears to be a natural approach; s/he conceals it from students, and from him or herself at times as well. The discipline-based paradigm is effectively a concealed weapon instructors can use to hide behind, one that is endorsed by the profession as the norm, the default paradigm. When the instructor uses it primarily for his/her own self-protective needs, s/he will be less sensitive to student confusions in learning new concepts or solving problems.

Now we extend the instructors' contributions to student poor performance into the interpersonal arena.

**Interpersonal Instructor Contributions**

*How much to give*

There is potentially an ongoing interpersonal struggle between the instructor and the student. The instructor ideally wants students to be interested in the subject, to act as junior colleagues, and to be open to learning. Students typically want faculty to be sympathetic, to be interested in them, and to be open to learning about them. An issue of *punctuation* emerges: who goes first? Must the instructor show interest in the student for the student to show some interest in the discipline? Or must the student show interest in the discipline before the instructor shows interest in him or her? When students are uninterested and anxious, they may express themselves in a variety of ways, including acting in a hostile and a derisive manner that generates a negative response from faculty members. And when instructors show little interest in their students in terms of their learning issues, that generates a negative response from students. Further, it has been noted that students avoid novel approaches to understanding concepts and solving problems, seek less help, and employ more self-handicapping behaviors when instructors do not attend to them individually, i.e. to their perceived inadequacies, fears, and hesitations (Turner et al., 2002). When this is the case, I suggest that instructors are missing an opportunity to join in the learning process, both about themselves and their students. The instructors' EQ determines much of how s/he handles this.

Some instructors respond to the issue of how much to give students with comments like : "I shouldn't have to teach them how to read; they should have picked that up along the way;" "no one taught me how;" "you either figure it out or you don't;" "I don't know how to teach it;" "it's not my job;" "it's self-evident;" "I don't believe in hand holding;" "they're just lazy and don't want to think about what they read..” They may become angry if they perceive that students want too much and they may object to students receiving more direction than they had. They may also find students’ attitudes toward and expectations about the course threatening, assuming a reverse halo effect – namely, that students will try to take advantage of them, dislike them, and think them odd for excelling in such technical disciplines.

Another issue common to the interchange is *projection*. Both instructors and students act as if they have the same resources, the same strategies, and the same non-discipline specific scholarly skills – often in spite of believing just the opposite. Behaving as if students have these skills misleads the instructor and can misdirect his/her efforts. Many instructors are unaware of any specific processes they use to solve problems, taking it as a fact that understanding the relevant concepts and a knack for word problems constitute the resource base available.

*Threat and self protection*

Yet another interpersonal issue is dealing with the threat of failure and its interpersonal consequences. Students may be fearful that instructors will ridicule them if they make so-called “stupid mistakes” while instructors may despair over the glazed look on students’ faces, indicating s/he has lost them as a group. Or the instructor may be worried that s/he won’t be able to answer students’ questions. The default position for both sides is (self) protection manifested in defensiveness.

Typically, students become derisive, bored, and withdrawn, and instructors answer by being arrogant, frustrated, and controlling. Self-protection represents a closed ABC stance for students and low EI for the instructor.

Certain students’ defensiveness may be provocative, resulting in a reaction from instructors in kind. When this happens, the instructor’s EI has indeed been compromised. But even while reacting emotionally internally, many instructors try to appear calm and unruffled. However, disguising feelings of frustration and/or anxiety transfers them onto more vulnerable “hosts,” i.e. certain sensitive students who end up absorbing them as if they had been theirs all along – a double dose of the painful affect not accounted for externally and thus internalized by the student (Kahn, 1996), a precursor to super anxiety – pending poor performance.

There are additional interpersonal concerns that predispose the instructor to self protect, stemming from how s/he anticipates being perceived. In the past, the instructor may have suffered socially, if branded as a “nerd” or weird for his/her ability and interests. In any case, when an instructor sets foot in a classroom, the spotlight is on him or her. He or she has to perform, to do a job. The sheer number of students can serve as a source of intimidation. Compound that with “baggage” from the past and one can certainly then anticipate a non-trivial level of tension and apprehension.

Lack of trust and perceived threat may follow from the vulnerability of both instructor, who has to stand and deliver knowledge to a large and often initially hostile audience, and from students, who fear failure and judgment. Most of them are worried not only about learning, but also about the instructor’s response to their performance. Typically, students are the more vulnerable party. A sarcastic or impatient instructor or one who lashes back at disgruntled students can be considered verbally/mentally abusive.

Should an instructor exhibit a low level of self-awareness, self-monitoring, and responsibility, i.e. low EI, in general, he or she is even more likely to "act out" when confronting potentially intimidating circumstances, in particular by choosing to protect him/herself. That implies that s/he is no longer open to learning – about how better to teach and about how to deal with his/her own issues.

*The power of meaning*

In addition to the protection of self, the instructor may also act out by protecting his or her **meaning-making schema**. These consist of first, beliefs, perceptions, feelings, assumptions, evaluations, attributions, and interpretations about the various aspects of shared issues, such as pacing of the course, out of class help, problem solving, and student performance; and second, their disposition – whether or not they are articulated and illustrated, or covertly held, or tacitly assumed (A.C.T.); and third, their congruence, i.e. whether what is espoused is consistent with what the individual really believes; and finally, fourth, their accuracy and effectiveness in the analysis of a situation. Constructed by both students and instructors, pertaining to the subject, the problems, student performance, and other issues, they present a potential interpersonal battleground.

Instructors have a special power that enables them to influence student meaning-making, the power of the *transference*. This term used regularly in therapist-client relationships, refers to the phenomenon of the client trying to win "oceanic" approval from the therapist by endowing him/her with symbolic parental status (Freud, 1959). This symbolized relationship also plays itself out in a more muted form in the classroom.

Student and instructor meaning-making interactions include conflict and collusion, especially with regard to attributions for poor performance. Students' vulnerability to instructor attributions, generally R.A.T.E.D., is a function of the strength of the transference, the disposition of the attribution (A.C.T.), and the level of concealment of the actual contributors. Covert attributions made by the instructor have the greatest power to influence students into assuming the worst, as they fill in the blanks with negative self-evaluations (Sankowsky, 2003). Tacit assumptions made by students prevent them from being open to alternative explanations. If vulnerability is high, students are likely to endorse the instructor attribution, whether or not it on target. This sets the stage for creation of anxiety (in the untrained group) and escalation (for math averse and math anxious students).

In any case, the instructor must manage the meaning-making process, however tacitly. Effective management of the process entails balancing advocacy of one's own view with inquiry into others'. When instructors advocate ineffective/inaccurate schema and/or fail to keep a balance with inquiry into students' schema, this constitutes mismanagement of the process and can be another contributor to any ongoing student poor performance.

In general, mismanagement of the meaning making process will be labeled *theoretical abuse*, a term originating in the psychotherapeutic literature (Basseches, 1997). In that context, theoretical abuse occurs when the therapist puts his or her need to make sense of treatment situations ahead of the client's. The suggestion made here is that this kind of behavior is relatively common in pedagogical contexts in subtle and disguised forms. Why is that?

There is a similar knowledge and power differential between therapist and client and between instructor and student, one that is necessary for a transference response. Thus, assuming an educational parallel, theoretical abuse in a teaching context can be defined as an *instructor's* attempting to satisfy his or her own meaning making needs at the expense of students, e.g. by imposing his or her schema on them. Imposition might include, but is not limited to, trying to convince a student that there is no need to worry and that there are simple ways to approach problems. Well-meaning instructors may deliver this message prematurely, not allowing students to air their fears and raise legitimate points. More generally than simply pushing their own views, theoretical abuse means failing to elicit, elucidate, explore, and extend student meaning making. Like therapy clients, students need the supportive probing of an expert who then guides them into positions from which they can make the right moves to solve a problem or to grasp a concept. If instructors do not take into account what the various aspects of the learning process mean to their students and do not help to surface tacit assumptions, the result is theoretical abuse.

Often this happens with the best of intentions: instructors believe strongly in their views of what has taken place and what should go on in the classroom, how the discipline should be organized and presented, and how students need to proceed to maximize learning -- while also discounting students' views in the process. An instructor may be not fully aware of the inadvertent power s/he has to quash student meaning-making before it is even articulated when rendering his/her opinions.

He or she may sincerely believe that his/her proposals are in the students' best interests. But s/he cannot fully know what is helpful for the student without getting to know the student's thinking and feeling processes. In fact, the worst case scenario is students' collusion with inaccurate and self-protective instructor meaning-making as a result of their vulnerability to the attributions s/he makes. Table 1 presents common issues, triggers of student anxiety/apprehension, situational ambiguities, sources of TA (= theoretical abuse), and then begins to suggest what can be done to avoid TA (to be explored fully in the second half of the article).

| **LEVEL** | **ISSUE** | **TRIGGER** | **INHERENT AMBIGUITY** | **SOURCE OF TA** | **TA DOGMA** | **INSTRUCTOR NEEDS TO** |
|---|---|---|---|---|---|---|
| *Content* how they grasp new concepts | Naïve theories and assumption of continuity | Perceived difficulty; being lost | Mental models for sense-making | Accepted meanings | This is what it means | Get in touch with students' perceptions |
| *Process* how they go about problem solving | Strategies re reading and process | Gap between easy & hard; verbal complexity | Resources needed; prompts | Instructor projection; not realizing the need for prompts | This is what you do | Get in touch with his/her processes & provide a transfer |
| *Interaction* how they view the relationship | Tacit IP expectations & management of the MM process | The transference | Punctuation: who does what for whom, and when; when is it hand holding? | No need to delve into IP issues in this content area | This is what you need and what I can do for you | Get in touch with IP process |
| *Performance* how they progress in short & long term | R.A.T.E.D. attributions | Errors, lack of progress | Responsibility; who's to blame for student failure?) | Bell-shaped or bipolar distributions as justification | This is where you stand; some just aren't cut out for this | Get in touch with his/her contribution and investigate their reasoning |

| | | | | | | |
|---|---|---|---|---|---|---|
| | | | (student, instructor, discipline | | | |

Table 1

**The Discipline**

Mathematics is commonly thought to be black or white with no fuzziness. If one can prove something, then it's right. There is only one right answer to a problem. So the field appears especially intimidating because it looks so unforgiving.

Yet, the roads to understanding concepts and solving problems are many and varied. To negotiate these journeys, the instructor can offer special resources or can claim that just as there is but a single right answer, there is but one resource package to it, requiring a simple explanation perhaps augmented with some examples.

The discipline has a structure, i.e. its internal logic, organization and developmental processes. At any stage, it is based on a set of "individuals," i.e. component pieces, and a network of relationships linking them. What remains hard for students to grasp, unless made explicit, is that the relationships can become individuals as part of the discipline's growth . For example, in calculus, one can focus on the squaring function as a relationship between numbers, the individuals in that context. Then, when the derivative is defined, the squaring function is taken as a unit, an individual, with the derivative as a relationship (between functions in general). Composition of functions is another example, in which these functions, originally perceived as relationships, are now treated as individuals.

In general, the structural issues that impact students the most are transitions (such as relationships becoming individuals) and coding. Also on the short list are application, representation, and universal aspects of discipline growth.

By a transition, I mean either a shift in focus, such as knowing when to stop visualizing as in the example in moving from the law of complements in probability to more complex looking laws; or a shift in perceiving constructs as individuals.

By coding, I refer to the perception of mathematics as a language, in particular, in the expression of equations. For example, in a linear programming problem, when given the information "there are twice as many economy tires as radials," (Eppen, Gould, & Schmidt, 1996) more than half of a management science class wrote the equation 2E=R instead of the correct E=2R. I queried the class to get at the reasoning producing the error. This process revealed that most students had *visualized* two groups of tires, the economy pile twice as big as the radial. They then juxtaposed the 2 and the E. The gap in reasoning, their leap of faith, occurred implicitly: had students articulated their thinking, it would likely have been along the lines of "the E pile is twice as big; therefore the 2 goes with the E." They needed to come to terms with this process in order to then fully accept the translation required to express the equation (Sankowsky, 2003).

Students do not typically realize except tacitly that much of algebra also requires translating to set up problems (existential quantifiers) or to express relationships (universal quantifiers). Were they to fully understand this on a meta cognitive level, their transition to calculus might be far smoother. The same applies in terms of making the leap from arithmetic to algebra.

Using a universal relationship expressed by an equation in a specific situation and given the numbers that accompany the situation requires the students to substitute these numbers for the placeholder letters. This we call an application. Again, for adult learners, meta cognition in this context is invaluable, especially when visualizing is not enough.

The rationale for eschewing the meta cognition resource is that many instructors believe that students need to have a firm grasp on content before they can make any sense of structure. I have found just the opposite to be true: to paraphrase Mary Poppins, just a spoonful of structure makes the content go down…"

Another example: for a long time, it never dawned on me that some students had no idea what they were supposed to do with a formula or what its expressive purpose was. Those students needed it pointed out that typically, the left hand side term is not known to us, but all the right hand side terms are and the right hand side itself consists of an arithmetical combination of said terms. Finally, by numerical substitution, the letters are replaced by the appropriate numbers. I found that some students needed to have all of this explicitly stated.

**Poor Performance and concealed contributors**

We have discussed the sources of math anxiety: students' shaky history in learning math, low instructor emotional intelligence, interpersonal battles over meaning and punctuation, and the nature of the discipline. Following each is the associated contributor to poor performance: from students' shaky past come restrictive untested, and tacit assumptions (R.U.T.) leading to ineffective automatic behaviors (closed ABCs); from instructors' low EI and the perception of student attitude and behavior comes the default discipline-based paradigm; from the interpersonal wars comes the battle over punctuation and the control of the meaning-making process; and from the discipline come structural issues, such as transitioning and coding.

Now we need to demonstrate that each of these contributes to poor performance and how it does so in a concealed way. An answer for all is the truncation of the resource base available or the mismanagement of process, of which students rarely have an awareness or a vocabulary to note and address. For example, R.U.T. assumptions lead to instinctive counter productive automatic behaviors effectively blocking access to resources needed. Instructor's acting out to defend him/herself results in adopting the discipline-based paradigm, but concealed as such because professional norms justify it; this is often not what students need, in spite of their colluding with that choice. Transference "strength" and not taking into account students' meaning-making robs them of exploring their own thinking, a valuable resource. This all happens so fast that its undermining role remains hidden. Finally, the discipline's structure contains transitions and features mathematics as language, aspects generally not made explicit and therefore concealed.

(Many poor performing students cling desperately and instinctively to the one approach they assume has worked so far for them and so they completely miss the necessary transitions in perspective and approach their more successful peers access, not even realizing such resources exist. When they recognize subsequent failure, should that occur, they tend to attribute it to lack of talent).

Each of these contributors is typically also concealed as noted and therefore undiscussable. Should poor performance result from the truncated resource base, its concealment and the concealment of it as a factor in the performance have great impact – especially since students are therefore unable to find any external locus to account for failure other than the difficulty of the discipline, a constant within this learning context, and accordingly, will be forced into attributing it to lack of talent, provided they have made a reasonable effort. A few will attribute the poor performance to random factors, but only if they are very well defended or very self-confident.

The talent explanation sends students reeling into an abyss of negative emotions. For in mistakenly assuming that others are just better at learning with the same resources, in particular with the same way of focusing on the material, they have inadvertently and unnecessarily restricted themselves from what they need and have paved the way for their own downfall. And that dynamic in itself is concealed. Left with this explanation and further internalizing the conclusive belief that they are inherently lacking in ability and therefore are relegated to immutable mathematical damnation, they are especially vulnerable to both anxiety (if they are currently either averse or untrained) and to super anxiety (if they are already anxious).

This brings us to the bottom of Figure 1. Anxiety then is likely to develop in math averse students and may escalate in those already suffering from math anxiety. Even the average student will likely become predisposed to anxiety if there is no accounting or rather a false accounting for the failure. For the math averse or math anxious student, this becomes a self-sealing amplifying or cyclical loop as the attribution feeds back to the assumptions the student already has, further entrenching them and making future work in mathematics even more threatening. Should the instructor endorse the view that maybe they aren't cut out for math, math anxious students will invariably go along with that assessment. They are then likely to pick up any unexpressed instructor fears and doubts, and suffer from super anxiety as a consequence; they are then predisposed to learned helplessness (Seligman, 1992).

Meanwhile, the instructor may feel s/he has no reason to question his/her approach. This is the case when the distribution of performance seems to fall within acceptable limits. That is, s/he is often able to justify the number of poor performers as statistically acceptable casualties and this further quells any anxiety s/he might have had.

## PART TWO: TREATMENT AND PREVENTION OF MATH ANXIETY

The key to turning around this formidable array of factors lining up to produce the anxiety is at once simple and complex. On the one hand, instructors who buy into the causes I have suggested can choose to act differently. On the other hand, changing one's emotional intelligence and one's reactions to provocative and threatening triggers is not usually simply a matter of will. So what is an implementable way to proceed? I will first focus on macro issues before turning my attention to micro maneuvers and a more complete answer to this question.

### Macro View

Focusing on a macro level first, I endeavor to establish a climate of respect, humor, interest, and connection in the classroom. This is consistent with Hatfield and Trujillo's perspective (1994) on re reducing anxiety. In my model, teaching needs to have an expanded multi-faceted orientation. These are the seven aspects I favor.

TEACHING AS COUNSELING

Having experienced students even at M.I.T. as fearful motivated me to delve into the role these generally destructive emotions play in the learning process. My own early jitters alerted me to the fact that instructors could suffer from painful affect as well. In both cases, I assume that fears inhibit learning. This has implications for teaching, at least for me: I found it impossible to ignore the degree of anxiety I observed and so deemed it necessary to address and deal with the underlying feelings. I needed to understand what psychotherapists did when dealing with their clients' anxieties as well as with their own. Could I transfer some of this to the educational arena?

Clearly, as we have already seen, a lot carries over. The instructor-student relationship also mirrors the therapist-client relationship in other crucial ways: among them there are common overarching goals instructors/therapists have for their students/clients, namely for them (1) to become liberated from seeing the world through one lens; (2) to become more self-aware; and (3) to realize, through critical thinking and emotional processing, opportunity and choice where before, there had only been automatic behavior.

Similar to a good therapist, the instructor needs to withhold judgment even when teaching subjects with clear answers. Instead, the instructor will be well advised to *investigate* students' errors and elicit their reasoning (this parallels therapists investigating clients' "mistakes" in life, i.e. maladaptive behaviors while delving into the underlying feelings). Second, premature advice giving or in general, doing "it" for them or telling them what to do robs the students/clients of the opportunity to grow by their own personal discoveries – another parallel. Instead, the instructor can provide resources in a timely way that allows students to make breakthroughs in their learning – just as therapists put their clients in a position to grow.

In addition, the instructor can function directly, albeit in limited fashion, as a counselor in helping students confront their feelings, focusing mainly on their apprehensions about learning a specific discipline. Just making these emotional reactions permissible to air and helping to make them explicit serves as a major ice breaker.

The interpersonal dimension of the teacher-student relationship generally does not receive the same scrutiny as the therapist-client dynamic. But, as noted earlier, theoretical abuse also flourishes in the exchanges between instructor and student. To avoid it, instructors need to be trained or train themselves not just to supply answers or correct mistakes: when students commit errors in problem-solving or exhibit confusion in confronting a new concept, the instructor needs to first get in touch with any frustration s/he is experiencing – quietly! – and then must adopt an investigative stance, exploring the students' mental processes as well as his/her own. The instructor should wonder, "what allows me to get this?" in addition to wondering what prevents them from getting it.

For example, the decision tree analysis referenced earlier requires the construction of a schematic that contains quantitative information in order to select a best strategy. By breaking down the problem solving process, I soon saw that many students could not even parse out the decisions indicated in the problem's presentation, let alone deal with the tree or the numbers.

So how could someone not see, for example, that a company's dilemma as to whether or not to bid for a contract (the JM company referred to earlier) heralds a decision point rather than signals an uncertainty? Through asking questions, I got one student to say, "But I don't know what they'll do, so it's an uncertainty."

That really struck me. I had never made explicit the fact that we were basing the analysis on the company's perspective. I just assumed this to be apparent. And if there were several companies, that became a bigger identification issue: we had better know for whom we are consulting! By looking into my own process and making a tacit prompt explicit, I then advised the student to ask himself, "is that up to them?" when encountering an issue identified in the problem to see if it was decision, uncertainty, or just fact/data. If it is within the company's control, if in fact the company has to act on the issue, then that it's dealing with a decision. If it is an issue whose determination would not be made until the company acts and is beyond the company's control, e.g. would a certain competitor enter the bidding, then the company would be confronting an uncertainty. That distinction proved helpful for a number of people. I was amazed, though, that I had to make it.

For another view of a counseling orientation, Freiberg (2005) taught a math anxiety workshop in which the first assignment was to write a "math autobiography." This helped the participants revisit childhood scenarios they could consider more objectively as adults. Many were able to pinpoint a moment when their confidence as a math learner simply eroded. In general, the workshop proved empowering, with other exercises such as confronting symbols and examining their "self-talk" (in the first case, the notation didn't seem so threatening when they saw it with fresh eyes and in the second, the self-imposed negativity was perceived as far more insidious – but correctable – than they had realized.)

Journal writing is yet another device used to stop the sense of falling behind (Stuart, 2000). Keeping a record of questions to ask in dealing with new concepts as well as preliminary thoughts about problems seems to help students feel more relaxed and more in control.

Vacc (1993) emphasizes the need for a process explicit approach, deviating from the discipline-based paradigm; he recommends customizing as well. Tobias (1993) and Greenwood (1984) point out the need for explicit problem-solving strategies. All of these can be viewed as counseling or training oriented as they deal with perception, affect, and behavior.

One of the keys to a counseling orientation is a dialectical relationship between errors and growth. Students need to experience that every error creates an opportunity to learn, provided the climate is one of acceptance (Barnes, 1984; Sankowsky, 1998). I try to demonstrate that the learning process is <u>enhanced</u> by mistakes, because they allow for necessary distinctions as the decision tree example demonstrates. I invariably will point out that students have done something correctly, even when they commit errors, because often they simply answer a different question than the one asked. Or if they try to extend a way of understanding, I emphasize that there was a need to make a transition, as noted earlier. Further, I share my experience of struggling with certain concepts, for instance the geometric "locus of points," which bedeviled me in 10th grade.

## TEACHING AS SOCIALIZING

As I spent more time with individual students and focused more on the process of getting at their self-imposed restrictive assumptions and self-defeating behaviors, I felt we were in this venture together; they had become people whom I cared about. I wanted to see them on a regular basis and class provided a natural setting for us all to get together 2 or 3 times a week. The group environment of 40 students differed obviously from the 1, 2, or 3 students I would assist in office hours, but the combination of connection with many of them individually and the potentially high energy level of the class as a whole provided me with another focus. I saw our coming together as a <u>visit</u> with me as host. We had an ongoing project that gave us the impetus to meet; in a figure-ground shift, the course sometimes would become a vehicle for socializing, albeit in a structured manner.

Taking on the role of host, I felt it important to maintain a spirit of friendliness and fun in the classroom. I had to think about how to keep the social aspects of our meetings alive and well. This influenced my planning of activities as I monitored the pace and rhythm of our ongoing academic conversation, e.g. the development of concepts and review of problems. I wanted the spirit of friendship between me and them, and within their numbers to make it more natural and easier to participate, especially for them to feel free enough to ask questions. Now, I spend 25% of class time with them in small groups (up to 5) to work on problems. Although hardly a novel idea, by emphasizing the social aspect of the project, again I hope that students will help one another with good fellowship. I encourage this in office hours as well, seeking to bring three students in at a time. This does not replace the one on one venues; rather it simply adds another dimension, enriching the social fabric of the class.

## TEACHING AS ENTERTAINING

An aspect of entertaining and important enough in its own right to deserve separate mention is the use of humor in the classroom. Not all instructors can be funny, of course, in the sense of joke telling and delivering one liners with the timing of a good stand up comic, but anyone can learn "process humor." This term (Ornstein & Sankowsky, 1993), refers to capitalizing on shared history, discussing the "undiscussable," personal associations, self-disclosures, exaggerated visuals, and situational peculiarities, just to name a few. After all, it's particularly difficult to find anything amusing about calculus, if one must take this course to satisfy a requirement. So if the content affords few opportunities for humor, we must look to certain processes linked to teaching.

For example, the class establishes certain patterns after a while and these become tacit, or at least part of the background. Making explicit reference to them on occasion seems to invariably lighten the mood, eliciting laughter. For instance, a student who often arrives late may draw a mock double take from me. I take particular pleasure in “responding” to someone’s cell phone ringing, announcing “I’m out of the office.” This not only is perceived as humorous, but unexpected since I don’t get flustered or angry at the ringing of the phone: of course it’s rude to leave it on in class, but I get better results both as a manager of the class (more in the next section) and as an entertainer.

To introduce humor in normally anxiety producing symbolism, I exaggerate to the point of being ridiculous. For example, with Bayes’ formula in statistics, I start with a division line that stretches from one end of the board to the other. In effect, I’m saying that I know this is how students see it and I join with them temporarily in experiencing “the big fright.” I then switch gears and become serious, focusing on structural issues, but this bit of humor, playing off the feelings typically triggered, often helps students buy in to “the math.”

Sometimes, I lapse into French for no apparent reason. This becomes an example of repeating a “bit” so that after a while it takes on structural meaning. Sometimes, I free associate and then remark that they as a group act as my therapist. Finally, when we go over problems in class, I find ways to have fun with the contexts.

## TEACHING AS MANAGING

The class can be viewed as an organization, even if only a temporary one (Mazen, Jones, & Sergenian, 2000). The instructor occupies the role of manager/leader in terms of determining what takes place, starting with the syllabus and continuing during classes. The potential exists for either an adversarial relationship with students qua followers or a cooperative one. The instructor faces a special challenge when managing courses that intimidate students – like the quantitative methods offerings analyzed here. This becomes even more pronounced when students also deem the courses to be irrelevant and thus of no interest. In situations like these, the manager must be a leader who somehow finds a way to motivate and transform the experience of his/her followers.

Accordingly, the instructor must manage the interpersonal “punctuation” – that is, which comes first, student interest in learning the discipline or instructor interest in learning about the student. Showing students that he or she wants to learn about them even while they remain guarded is almost always helpful and an easy choice for the instructor interested in getting a deeper understanding of the learning process. As noted, it moves the relationship in the right direction, generally enhancing student motivation to learn.

The real dilemma instructors as leaders face is how much to do for students, in terms of resources. Should certain resources be withheld, e.g. special software or instructor time/directives, because instructors fear that students will abuse them by reducing effort; or should they be given freely as students argue for their legitimacy? In a cooperative relationship, trust carries the day: the instructor as leader sets the tone and uses his or her judgment as far as how much to provide. If trust has taken root, students will likely believe that the instructor acts in their best interests. Some of the other aspects of teaching referenced earlier can help establish trust.

On the other hand, nowhere is the potential for an adversarial relationship to materialize greater. Without solid trust, each side tries to protect its interests and typically marshals seemingly objective rationales with respect to the resource issue. And yet it may all come down to a disguised struggle over control: “do it for me, this is too hard” v. “this is as far as I go, you have to do the rest on your own – in order to learn." This kind of interaction occurs when either or both sides in an organizational context feel threatened (Argyris, 1990).

This issue can be viewed from the perspective of instructor emotional intelligence: high EI is associated with trust and low EI with adversarial relations. Very often, events in the classroom trigger feelings that propel instructors toward "acting out" – that is, to suspend self-control in favor of an immediate comeback, often over reacting in the process. A high EQ will serve the instructor as s/he resists the automatic response the trigger calls for.

TEACHING AS LEARNING

As I have indicated, I need to learn about my students – not only to motivate <u>them</u> to learn, but for my own growth as an educator. I need to know how people learn and what gets in their way when they don't. To do so, I engage in self-study and in studying them. I need to understand what I do, how I look at the material and what prompts I need that makes life in the math lane a 65 mph breeze; and I need also to have a sense of what anxious students do, how they look at the material, and what prompts they apparently aren't getting that makes their life in the math lane total gridlock. For example, when students come for help in office hours, I invariably find a little unexpected wrinkle when I go over the same ground as I do in class; this wrinkle tells me more about the way people think about the concepts, symbols, and problems. Sometimes, I feel a bit of frustration rise to the surface when I can't get through to them. I realize that's my issue and work on wrinkles in my role as provider.

At times, the cumulative effect of many wrinkles leads to a fundamentally new grasp of the educational process. In effect, we have partially switched roles – they teach me, or more precisely, I learn from them. I learn what works and what doesn't. The more open I can be, the higher my EI becomes and the more I am surprised and able to truly discover something new about my profession; thus, engagement and discovery empower me. They also promote a sense of partnership, which empowers them as well.

All of these measures help establish a non-adversarial environment. It remains to be seen how we can implement them to effect the changes.

**TEACHING AS EDUCATING: The development-based paradigm**

I have saved the most obvious aspect of teaching for last, positioning it as a launch point for a student-friendly alternative paradigm to the content-only approach. The ultimate aim for me as an educator is to go beyond the discipline, using it as both important in its own right, but even more so as a vehicle for studying learning processes in general. Math is ideal as a "laboratory" for surfacing these issues. All of the teaching recommendations mentioned so far are subsumed under the development-based paradigm.

**A new paradigm**

The development-based paradigm of instructional delivery serves as an alternative to the discipline-based (Sankowsky, 1998). It has as its vision the value of scholarship and learning in general. Its mission is to explore the learning process, i.e. to focus on underlying and universal aspects of discipline growth, to view the discipline as a vehicle for developing precise thinking and as a vehicle for increasing the student's self-awareness as a learner. Its implementation requires focusing both on the discipline's "infrastructure" and the psychology of learning as well as the mastery of content. The role of the instructor shifts to that of facilitator and investigator of student learning. Students' objectives now include growing more self-aware as learners and using the course as a vehicle to become more precise in their thinking in general.

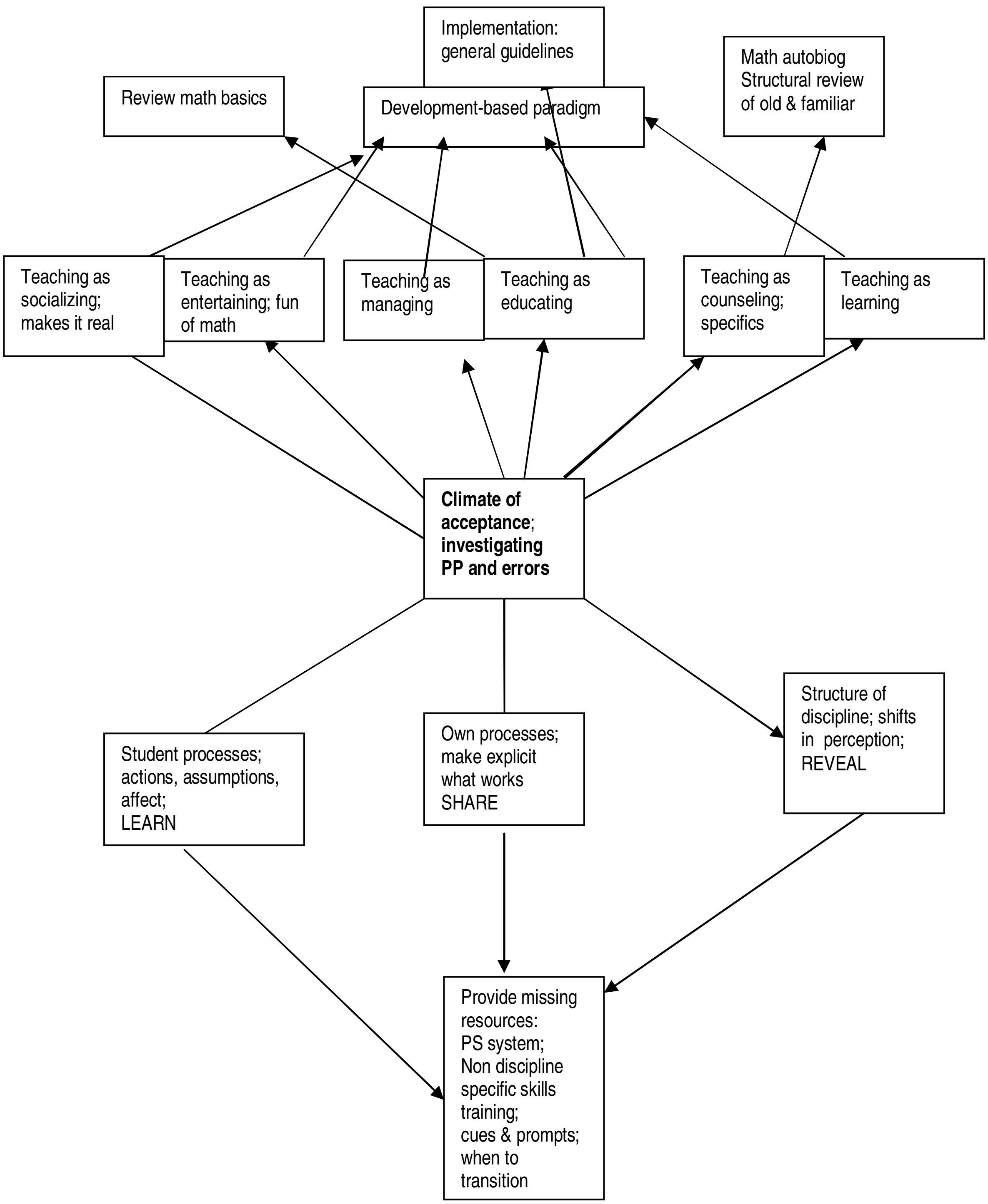
The development-based paradigm appears to carry enormous costs. First, it typically does not occur to an instructor of, say, statistics, to even consider these (process-based) issues. Second, once made explicit, the development-based paradigm would appear to take the technical instructor into unfamiliar territory, requiring counseling sensibilities and training beyond reasonable expectations. Third, such an approach would seem to divert energy and time away from standard content coverage, constituting an investment whose payoff is unclear.

But the benefits side can address these concerns. First, apprehension in the classroom is real and harmful to learning unless addressed in some fashion (a good discipline-based provider can do so without making that issue explicit, preventing some of the anxiety through clear and well-paced presentation of content). By explicitly mentioning the ABCs, instructors acknowledge/defuse some of this tension, and students are given an alternative attribution for poor performance, one that provides some degree of hope. Second, a student-centered presentation that makes explicit reference to non-discipline-specific infrastructural issues, such as the purpose of a concept, the reasons for the symbols used to convey it, and the process of numerical substitution for computation will clear away many student assumptions and make them more aware of their existence in the first place. Third, by aiming for a "higher" goal, i.e., the discipline as a vehicle for mastering precise thinking and articulation, instructors may tap into student intrinsic motivation to learn. This may make the investment pay off in the short term, i.e., within the course itself, for three reasons: student energy and motivation are mobilized; instructors themselves are learning, challenging themselves in the process; and the initial focus on infrastructure and psychology, although appearing to be time-consuming, actually can be done fairly efficiently, and allows for accelerated coverage of content later on.

**Micro implementation of the development-based paradigm**

To implement the development-based paradigm, we need to have a sense of which of the concealed contributors to address directly, allowing others to fall into place as the instructor finds value in locating them. Micro techniques provide a way to reveal the discipline's structure, probably the easiest entry point. From there, student counter productive tendencies and confusions over concepts begin to bubble up. The instructor then is in a better position to self-study with respect to the discipline and share heretofore underlying strategies and make them explicit. Summarizing, s/he aims to **learn, share**, and **reveal**: to learn about what students do and how they focus that sets them up for failure; to get in touch with and share what s/he does and how s/he focuses that allows him/her to succeed; and to reveal structural aspects of the discipline that open up conceptual vistas for students. This demystification process does not require a counseling background nor even a high level of emotional intelligence – at least at first. By beginning with the discipline and agreeing to investigate elementary structures while withholding judgment about the students' need for explicit mention of these issues, the stage is set for much of the rest to follow. And those two actions/attitudinal changes alone will make quite a difference for many students.

Figure 3


Implementation: general guidelines
Math autobiog Structural review of old & familiar
Review math basics
Development-based paradigm
Teaching as socializing; makes it real
Teaching as entertaining; fun of math
Teaching as managing
Teaching as educating
Teaching as counseling; specifics
Teaching as learning
**Climate of acceptance**; **investigating PP and errors**
Student processes; actions, assumptions, affect; LEARN
Own processes; make explicit what works SHARE
Structure of discipline; shifts in perception; REVEAL
Provide missing resources: PS system; Non discipline specific skills training; cues & prompts; when to transition

*Making the New Familiar and the Familiar New: Connections*

Every new construct can be introduced with examples. Equations and other general conceptual aspects should be initially withheld. Examples from both the quantitative and non quantitative realms can be used. The message to be conveyed is "you've been there and done that, but just don't know that you have." To properly communicate this perspective, we must make tacitly known material explicit within the examples. And we should begin with specific details of a situation, within which the concept is embedded and from which it can be parsed out.

In calculus, the central and challenging notions of limit and derivative certainly seem remote at first. Instructors can announce that students already have familiarity with these ideas, at least tacitly, whenever they look at the speedometer in their car. By fleshing out the notion of instantaneous speed at a moment in time and distinguishing that from average speed over an interval of time, instructors are making the new familiar. They are paving the way for introducing the derivative in this context. That is, they will use limit in a sentence, to wit, the instantaneous speed is the limit of the average speeds over progressively smaller intervals, length approaching 0. Instantaneous speed is also the rate of change of distance, or the derivative of distance with respect to time. Augmenting this presentation is the use of numbers in the form of hypothetical data to illustrate, e.g. seconds, tenths of seconds, hundredths of seconds, and so on. (The sequence 1, 0.1, 0.01, 0.001,… seems to be heading to 0, and this can also be used as an example of limit).

Going further with this example, constant speed is seen as a special case in which average and instantaneous speeds coincide. On the other hand, with objects dropped from a height, the speed is not constant, but there is a <u>formula</u> for distance fallen as a function of time. We can then demonstrate how to establish this formula with a minimum of algebra.

To see how to use a non-quantitative example, consider the concept of function. We can invoke "birth mother" as archetypal: everyone has one and only one birth mother; who the birth mother is <u>depends</u> on who the person is; specify the person (first) and you specify the mother. The accompanying notational change makes the familiar new.

For a very simple non-quantitative example of composition and inverses of functions, let A = putting on socks and B = putting on shoes. Then B o A can be discussed and its inverse seen as $A^{-1}$ o $B^{-1}$ first taking off shoes and then taking off socks. And this is a great example of non commutativity of operations as well as iteration of operations: for A o B means putting socks over shoes while A o A = putting on two pairs of socks.

Using the abstract functional notation when it is not necessary because the situation is so well understood makes a case for investing in it <u>before</u> it is really needed -- besides building bridges between new (concept of composition of functions and the symbols) and familiar (the specific example).

A similar investment introduces the infamous x that plagues algebra students in arithmetic – as in x = 5 + 3. It's not necessary for the computation, but it makes one aware of the communication and the concept of problem statement as separate from problem solution.

*Noting Transitions*

Every construct comes in a conceptual, communicational, and computational package. That is, one can choose to focus on one aspect at a time and to then to attend to the others. That's one of the transitions.

I like to point out that students have made such transitions in perception and focus earlier in their math lives, hoping to connect them to the fundamental similarities between the new concept or process they encounter and familiar ones. Generally, the first such example where this occurs is in learning the concept and representation of a specific number, say 3. Even though there is a transition between grasping 3 sticks or 3 airplanes and the notion of 3 as in the sentence "this is the third time you've said that," they have made that transition, albeit tacitly. I make it explicit and hope they will get the connection.

The tire example given earlier (twice as many radials as economy) calls for a transition from visualizing to translating. With this general issue now discussable, students can learn first, that one cannot always visualize, and that expressing equations is an example of the language aspect of mathematics.

Back to calculus: with the derivative, we can talk about functions and their rates of change on a purely conceptual level: speed – distance, cost – marginal cost, money – growth. Then we bring in notation: D and D'; D(t) = . Finally, we focus on computation as in the free fall example.

Another transition is from context-based examples to an abstract construct. The structural shift from "3 cows" to the number 3 provides an example in a very elementary setting. In calculus, we talk about a particular application, say D, and then move on to talking about functions, F, in the abstract.

We also shift focus from individuals to relationships between them, universalizing in the process. As simple an example as noting that anything plus zero is that anything moves us from the concrete $3 + 0 = 3$ to the universal $x + 0 = x$. We can point out that x is a universal placeholder in this equation, translation still required. Non quantitatively, we parallel this move from the concrete "John shaved this morning" to the universal "everyone on the bus shaved." Notice that we can continue to link to the familiar, making tacit knowledge explicit in the process.

A perceptual transition takes place in moving from visual/verbal to pattern recognition. For example, in probability as noted earlier, the law of complements, $P(A')=1 - P(A)$ is relatively easy to grasp visually and to express verbally (probability of an event's opposite is one minus the probability of the event). Another law involving conditional probability, is not so accessible visually and verbally: $P(A \cap B) = P(A|B) * P(B)$. This relies on pattern recognition and on identifying clusters of terms.

The zero-th power of a number, say 10, yields another example. Students often guess 0 since there are no tens to visualize if, for instance, they have understood $10^3$ as 3 tens multiplied together. To grasp $10^0 = 1$, it is necessary to see it in relation to $10^1$ and to realize that the next power of 10 is 10 times the current power. So to maintain this relationship, it should be pointed out to students that we <u>define</u> $10^0$ to be 1.

One might object and claim that visualizing is still possible for $10^0$ ; if $10^4 = 10000$, then the 4th power can be construed visually as 1 followed by 4 zeroes; so the zero-th power should be 1 followed by zero zeroes! Or just 1.

But when it comes time for $10^{-1} = 1/10$, visualizing, while still possible, becomes much more arbitrary; the maintenance of the relationship takes over as the more economical explanation. The number -1 is itself defined by $x + 1 = 0$ (an additive inverse).

Another perceptual transition takes place when we move from focusing on combinations of symbols as interactions between individuals to then perceiving them as single (sometimes higher level) units. In an elementary quantitative context, we look at 3 + 4 as a sum but also as 7. In a non quantitative context, Dan's mother is seen "in relation to," but she can also be seen as Sue, an individual.

*Discussing Universal Aspects of Discipline Growth*

We focus on five growth generators that we can discuss in context: reorganization, distinction and reconciliation, relationship preservation, (two step) reduction and resumption, and extension. These processes are routine incremental steps that lead to discontinuities in discipline development through "punctuated equilibrium" (Gersick, 1991).

Reorganization can be exemplified in statistics in the concept of a distribution. By simply putting values of a variable first and then looking at their frequency, as opposed to listing observations in the order recorded, we take a leap forward, with a powerful new idea. In calculus, reorganization on a symbolic level takes place when we introduce the prime notation for derivatives. Besides being symbolically economical, the prime notation allows us to grasp the second derivative more easily.

Distinction and reconciliation lead to the concept of limit in calculus. With distance traveled as an example, instantaneous speed is first differentiated from average speed. Then the two are reconciled in the special case of constant speed. This makes for a passage to the approximation of exact rate of change at a point from average rates of change over progressively smaller intervals.

Preservation of relationships helps generate new individuals. For example, as noted, the 0th power and negative powers are defined as whatever number maintains the rule of addition of exponents.

The two-step approach to growth means reducing complexity to define one concept/build one aspect of a theory, and then reintroducing complexity at a later stage – by relaxing assumptions necessary to build a theory or by looking beyond the simplified special cases. Probability and statistics reveal a two-step separation of this nature. First, we assume we have population data and are able then to uncover various probabilistic relationships. These relationships form the basis of statistical analysis, when it is no longer assumed that we have access to the population.

Extension refers to attempting to duplicate a procedure or a conceptual development, leading to the creation of a new entity. For example, in statistics, the Z distribution is used in confidence intervals and hypothesis tests when the population standard deviation is assumed known, or when the sample size is large. For small sample sizes and unknown standard deviations, the t distribution is developed, paralleling aspects of Z. Then, the concept of a "statistic" is born and the gates are open for chi-square and all the non-parametric tests.

*Labeling Student Activities*

Students tend to be busy performing actions rather than reflecting on them. By providing them with a vocabulary to stop and label what they are doing, we take a step to helping them rebalance between action and reflection. At least the following student activities can be so labeled: recognizing in context, abstracting/universalizing/applying, translating, interpreting, reasoning, self-prompting, and balancing. And there is computing, communicating, and conceptualizing as well.

*Recognizing in context* is like trying to find one's car after coming out of a mall whereas parking it is like grasping a concept. In other words, even if students understand what something means and even if they can provide illustrations for that understanding, they may not be able to pick it out in context. They need to ask themselves, "what does A mean in this setting?" and "what does B represent, in terms of a possible A?" For example, in queuing theory, students may feel comfortable with arrival rate as an abstraction or when the population is people (as in prototypic examples), but may still fail to pick out "machine break down" for instance, as another application.

*Abstracting* refers to removal of a context – as in going from 3 cows to 3. Universalizing refers to articulating patterns – as in $x + 0 = x$. Applying refers to specifying from a pattern: so $7 + 0 = 7$.

*Translating* qua activity takes place when we deal with symbolism. To go from English words to an equation requires that we translate. Conditional probability provides an example: $P(O|L1) = 0.44$ is the symbolic equivalent of the verbal "44% of type 1 land is oil-bearing." In this situation, the conditional is signaled by the English "of"; other times, "if" and "when" are used.

*Interpreting* refers both to making verbal sense of equations, including a discussion of their significance/implications, and also to overlaying translation with recognition. For instance, to know that a conditional is required in the oil-land statement, students need to go beyond the verbal-symbolic connection to a conceptual one. Another example of interpretation comes from printouts on regression in which the Coefficient X1 number is put in a sentence: in an analysis of salary of executives in a company that awards merit points, the number 47 in the coefficient one slot can be interpreted as "it is estimated that mean salary increase goes up $47 for each additional merit point."

*Reasoning* means that students must fashion equations and the like from certain basic relational building blocks. For example, if each pound of an ore costing $4 yields 0.7 pound of final product and that product sells for $10, students need to go through a process, deductive reasoning, that allows them to see that the revenue from a pound of ore as input is $7 and the unit profit from the ore is then $7 - $4 = $3 /lb.

*Self-prompting* occurs when students ask themselves questions and use various cues. For example, if they are told that 34 out of 200 wells in this region are oil bearing and then asked for the probability of finding oil, their computational process is aided by focusing on the 200 and the 34 and asking themselves, "out of all (200) wells in this region, how many (34) had oil?"

By *balancing*, I mean that students look both inside and outside a given piece of information. That is, when confronted with details, they need to retain the right to say to themselves – from the outside – that's a set of details about XYZ, in addition to looking inside the details (which they tend to feel compelled to do without outside intervention). To augment their sense that they can wait before dealing with information, students need to balance categorization with manipulation. That is, it may be helpful at various stages of a development or a problem, to simply note and store the information as such – as a decision, as an uncertainty, as a probability (to recognize it for what it is in general). Faculty can help students by sharing that they do this all the time, and that students can learn to do it, even if it may seem a bit unnatural at first. The classic example is that of someone who provides an argument in a case; students who lack the appropriate balance respond immediately with an opinion, a counter argument, etc. Those who are more likely to have some balance may simply note that an argument has been presented as an initial step.

<u>An illustration</u>

Consider the concept of conditional probability. Instead of using a law to define it, we implement the development-based paradigm as follows.

*Begin with an example*, setting probability in a context familiar to any adult learner. Consider 1000 homeowners in a region outside Boston, made up of 6 cities. Suppose 456 take public transportation. Probability in this context can be defined as a percentage: P(T) = 456/1000 = 0.456 (=456 OUT OF 1000). On a conceptual level, it is the link between individual uncertainty and collective percentage, connecting the existing collective with any future one. On a communicational level, it involves the use of functional notation, which can be discussed. On a computational level, it is just a matter of "doing" percentages as fractions, with "out of" translating to "divided by."

Conditional probability is then introduced by showing a contingency table with the towns as columns. Suppose 268 homeowners come from city 3. We want to know the probability of a town 3 homeowner taking public transportation. The concept is still individual uncertainty linked to a collective, but the collective has been updated to just the city 3 folk. To reflect this in communication, we need an additional symbol since P(T) refers to the whole collective. For the reduced collective, we use P(T|C3), reading it as the probability of taking public transport for someone from city 3. Computationally, we adjust by using 268 in the denominator and ask: "out of the 268 people in city 3, how many take public transportation?" If that number happened to be 164, for example, then P(T|C3) = 164/268.

Then conditional probability can be formally defined, still without using the associated law with joint probability. Other examples can be given, some without numbers, such as in the case of a trial: G = defendant guilty and J = jury finding guilty. Then P(G|J') = the probability of an acquitted individual (actually) being guilty while P(J|G') = the probability of an innocent defendant being found guilty.

Here we see making the new familiar, bringing out tacit knowledge, revealing transitions (the 3 C's, context-based to abstract), and labeling student activities (recognizing; translating; acting on prompts).

*Problem Solving*

Steps to handle complexity: the 4I's and more

Students need specific resources to deal with complexity on perceptual, motivational, and behavioral levels. First, they need the 4I's: information, identification, intervention, and internalization. By "information," I mean understanding some of the issues that call for change/growth (e.g. the tendency to over visualize) and having some sense on a cognitive level of the processes involved. By "identification," I mean recognizing how they apply on an individual level (this is how I make mental pictures, for example). By "intervention," I mean allowing someone else (or even oneself) to design an implementable shift in either the cognitive, affective, behavioral or perceptual dimensions at a critical point within a problem-solving venue. By "internalization," I mean being able to recognize and choose when to make the shift on one's own, so that eventually it represents one's new "standard reaction." We could rephrase all this as "knowing people have problems"; "knowing I have a problem and what it is"; "experiencing that problem in context and seeing a fork in the road"; "following the other fork."

*Information* breaks the silence and acknowledges the elephant in the room: the tension that exists between quant jocks and quant phobics, the fact that the R.A.T.E.D. attribution is typically invoked and the reaction students have to it. I share with them my feelings about my own difficulties with certain subjects and how I too could get down on myself. I inform them of common counter productive problem-solving approaches, indicating that they may benefit from learning which apply to them and in what form *(identification*) and how they can make a shift to a more productive approach (by allowing *interventions*). One such global intervention is the adoption of a multi-stage approach to problem solving. It all starts with reading.

3M approach to reading

To slow down those who try to solve the problem as they read, to give the ones who read and reread the same way each time a variety of approaches, and to provide the students who can't get started with a routine way of cutting details down to size, I suggest the 3M strategy: meta-cognition, multiple passes, and matrix identification.

Meta-cognition requires a structural vocabulary through which students learn to perceive information "as," i.e. to categorize the details rather than feel forced to deal with them right away. The qualitative interpersonal equivalent is pausing and verbalizing "what are we arguing about?" as opposed to continuing to argue. As noted earlier, it may be the case that sometimes going outside/beyond content at the start helps an individual see that content more clearly, provided transitions are smoothly mapped out. Thus, they may need to go meta first and then come back to content.

Another aspect of having a meta-vocabulary is that it allows students to label any confusion they experience, guarding against unnecessary escalation into anxiety and setting up "early interventions." It can help them feel more in control of themselves and the discipline. Finally, it provides a way to stop doing and start reflecting.

Multiple passes refers to rereading several times with different goals for each read. This strategy seeks to set up interim goals and accordingly, to narrow one's aim during a given reading (widen the focus but narrow the goals). Its intent is again to help students set aside details at the initial stages by providing an alternative focus. For example, in decision tree analysis, I recommend three passes at the written information. The first one is a quick scan that aims to pick out the companies involved and identify the one "of record," i.e. from whose perspective the information is to be viewed. The second read looks for a broad overview of the dilemmas and prospects the company has to deal with. The third goes phrase by phrase throughout the problem, identifying each qualitative fragment of information as either modeling, uncertainty, or decision, ignoring all other information for now. This is basically meta-commenting; I refer to it as the MUD list. For this read, I instruct the students to disregard all the numbers they see (except non-computational ones such as dates, quotas, etc.). They should reread the problem to access those numbers once the tree is in place.

Matrix methodology means coming at concept identification from two directions. For example, one can label a phrase such as "each compact engine block requires 7 hours of milling time" as a unit quantity – stripping away any other details, in a read that has as its goal identification of information (meta), seeing "as." On the other hand, students can engage in another read, where the goal is to identify the unit quantities. So, the first read takes the information and asks, "what does it represent?," with a structural vocabulary; the second approaches the information and asks "how does one apply this concept (unit quantity) in this problem?" Having two ways to get at the same issue makes it more likely the students will grasp it and also helps them appreciate the distinction between application and representation. Having them use these terms reinforces their familiarity and comfort with a meta vocabulary.

The 3M approach thus offers the following resources: a meta-vocabulary that provides a *structural* analysis; a multiple-goals orientation that provides a *process* for reading, dealing with counterproductive tendencies along the way; and a matrix-recognition organization that provides more robust *connections* between concepts and examples, in particular, enhancing application and representation skills.

Focus on structure

Structural analysis takes content of the discipline as object rather than subject: it assumes some knowledge of the relevant constructs within the discipline and then erects *taxonomies* (typing concepts), pinpoints *distinctions*, "unpackages" *information* (e.g. breaks certain concepts down into usable fragments), and reveals *transitions* (e.g. from individuals to relationships; in how to focus on information), as we have just seen. It requires a meta-vocabulary to separate it from the actual content – as it discusses content without being content. It provides students not only with a useful analytical apparatus, but it also shows them a way to slow down and reflect more effectively.

### *A multi stage problem solving system*

The problem-solving system is introduced during the first week of the course, but students don't typically see its value because they have not used it; some ignore it completely because they're not graded on it.

I present a system with the following features:

1. A set of interim goals (rather than have them aim for an immediate solution)
2. Routinely implementable steps to achieve these goals
3. A set of self queries and cues to make the steps routinely implementable
4. Information about the process – such as the tendency to over visualize – to help them accept the self queries and cues

The system is labeled RTCI, which stands for *reading*, *translating, computing*, and *interpreting*. As we have seen, the reading section is a major part of the training and has been discussed in detail. I explain the other stages as follows:

Translating, the second leg, means substituting symbols and diagrams in the place of words. It's like learning how to use a shorthand, a new language. The symbols are designed "aerodynamically," that is, to express relationships and concepts more economically, more explicitly, and more precisely than words. For example, conditional probability is indicated verbally in many different ways (if, then; of all; or by juxtaposition, to name a few), but has a single meaning and symbol. During the translating stage, one chooses symbols for the component pieces identified in the reading stage, using these and relational symbols provided for a given topic in order to put various verbal statements into symbolic code; and one does the same for the question posed by the problem. Sometimes, as is the case with decision trees, translating means uses a schematic – the tree itself, illustrations forthcoming. Performing this function sets up the next stage, commonly identified as "the math" since it involves computation.

Computing is highly overstated as a fear factor. When reading and translating have put students in a good position, using Excel or a formula or writing an equation is almost "like butter." Prompts are given to help students get to that position.

Finally, interpreting means taking the results from the computing stage and saying what they mean, i.e. putting it back into words, and also spelling out implications for the company we are "consulting for." This as an exercise in communication, assuming that company decision makers know nothing of the techniques and framework needed to produce results – no jargon allowed at this stage! Templates will help accomplish this in certain instances, utilizing cues and prompts.

Such a system will eventually force students to come face to face with their counterproductive tendencies spelled out in part 1. At the same time, the system provides a viable alternative. It is a powerful resource, an external aid to help students organize information; such a resource is called "ideational scaffolding," a concept introduced by Ausubel (1968), It also helps them to represent the problem correctly; if they do, they markedly increase their chances of solving it (Rittle-Johnson et al., 2001). The system is designed to enable them focus on underlying principles and problem categorization rather than details, a benefit also suggested by Owen and Sweller (1989).

Four imperatives go with the system:

1. Note quantitative details as such, but do not act on them until a clear conceptual path has emerged; i.e. don't try to solve the problem while reading it
2. Categorize details on the first full read, but do not manipulate them

3. Widen the focus to include other problems
4. Link to a conceptual framework

By not trying to "get it" in one reading and also hold all the information in their heads, students become more patient and avoid the inevitable frustration of attempting to do too much. By narrowing the aim while widening the focus, they adopt more realistic and differentiated goals. For example, as noted, the second read through attempts to get some general understanding of the problem's component pieces and relationships, e.g. realizing that details may obscure a simple idea such as a company makes 3 products, using certain raw materials, with some flexibility in how it does so, rather than take note that mortar is 10% sand. Second, once having broken up the reading process with actions based on the relevant conceptual framework, they can then go back at a later time and read for more detail, having provided a "place" for some numbers and words. Finally, they can reread once again to test their direction and to insert numbers in the appropriate slots.

By holding off on the quantitative maneuvering, students lubricate the computational process. They do not try for too much, but rather wait until they are ready for this kind of detail and technique. In linking the quantitative to the qualitative, they work the computations within the boundaries of a conceptual framework.

By widening the focus, they are able to take advantage of their experience and compare the problem on which they are working to others. They can learn to see details without doing anything with them except to *categorize* them, by "backing away from them" and going "meta." This enables them to do what experts do, in finding several micro connections on a structural level to several problems rather than search for one macro connection on a superficial content level.

By linking to conceptual frameworks while reading, students take an active posture. They can ask themselves questions, reflect on what they have read, and then pick up on cues. They will not only say, "this reminds me of that" but will back away from detail as necessary, allowing the concepts to point them in the right direction.

<u>An example</u>

In 1978, I had to decide how to come to work. I saw only two choices: drive or take the bus. This introduces the concept of modeling: the fact that I, as the decision maker, will only consider the two ways of getting to work is something that the students, as my consultants (whose role is to construct the tree and report back on the "best" course of action), have to take into account -- even if they wonder, "What about the train, boss, the train?" Another bit of modeling is that the decision is to be made on purely economic criteria – this holds for all the examples we will consider. Finally, I ask that we model for a "regular day" (no accidents, breakdowns, etc. for either vehicle).

Now we also include monetary information. The bus is $3.40 round trip. Driving costs $1 in gas and $1 in tolls. Other costs depend on what happens.

My first driving concern is whether I find a (legal) space on Beacon Street. If I do, I will feed the meter ($.50 for the time I'm in town). If I don't, I have to decide whether to try to park illegally or simply put the car in the Common Garage ($4).There's an 80% chance of finding a spot on Beacon; a 90% chance of finding an illegal spot if I don't – so I'm told. If I park illegally, I run a .3

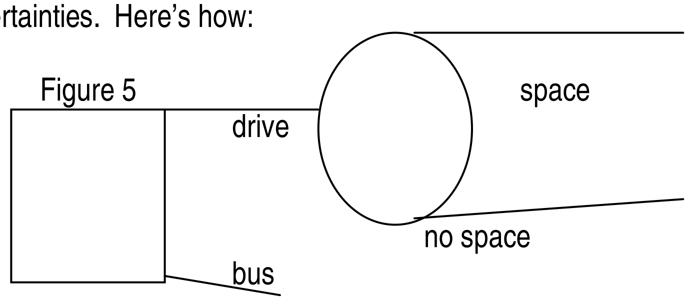
probability risk of getting a $10 ticket. How are we going to take all this information and make a decision that optimizes the economic consequences?

To see how, we have to build the tree, initially leaving out all numbers – that is, probabilities and monetary outcomes. The tree is a schematic of the chronology of all combinations of decisions and uncertainties – all pathways. These are called branches. I know I'm at the end of a branch when there are no more decisions or uncertainties to consider and I can specify the monetary outcome, the sum of all revenues accrued along the way minus the sum of all costs incurred along the way. Let's put some life into this by working out my tree.

*Building the tree*

The schematic is quite direct: squares announce decision points and circles announce uncertainties. Here's how:

Figure 5



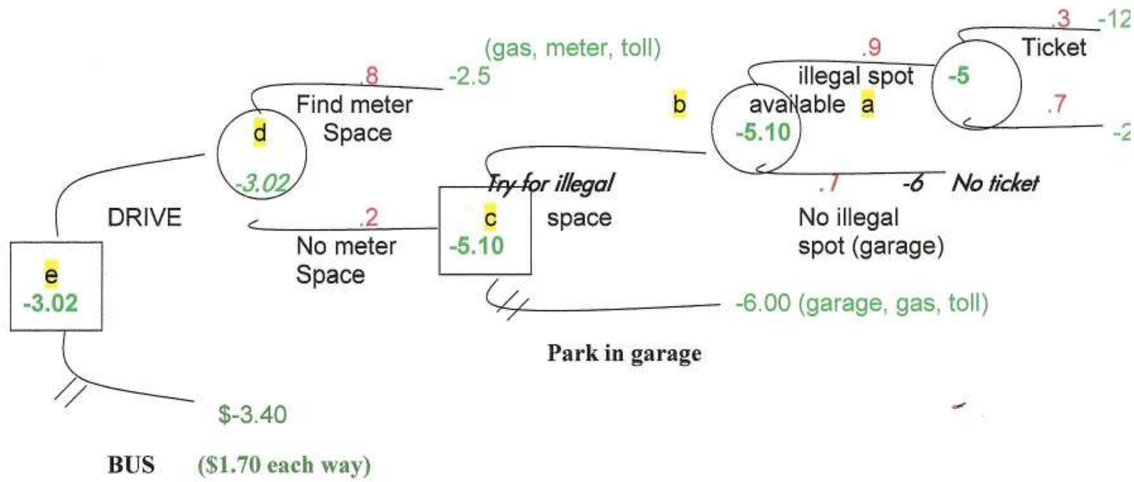

The choices associated with the decision are shown as lines coming out of the decision box, with the appropriate labels beneath them. The possibilities associated with an uncertainty are shown as lines coming out from the uncertainty circle, with the appropriate labels beneath them.

What we have so far is the beginning of my dilemma. I have to decide between driving and taking the bus. If I take the bus, there are no further decisions or uncertainties – assuming a regular day; I can specify the monetary outcome, -3.40. This is a cost, round trip bus fare, so it's a negative number: at the end of the day, I'll be out $3.40. On the other hand, if I drive, the next concern – assuming no accidents, speeding tickets, car jacking, and breakdowns is the uncertainty of Beacon Hill parking availability. That's represented by the circle, with the possibilities "space" and "no space."

If I find a space, I can specify the monetary outcome, assuming I feed the meter. So that branch is complete. If I don't find a space, things become interesting. Now it depends on what I would consider doing. If I were my father, I would automatically pack it in and head for the garage; that's his model and then the tree would be complete. However, I've lived in Boston long enough to entertain two choices: the garage and illegal parking. I should make that trying to park illegally, because there has to be some physical room next to the curb to put my car – and all spaces, legal or illegal, might be taken. My cousin, on the other hand, is so brazen that he would even double park for several hours.

Taking the tree from that point, we put a box after "no space" with lines "garage" and "try to park illegally." With "garage," I'm done. With "try," we confront the uncertainty of whether or not there is an illegal space. If none is available, I'll head for the garage and that branch is complete. If I find a space, there's a chance (uncertainty) of getting a ticket. So we would draw a circle with lines representing “ticket” and “no ticket” coming from it. The tree appears on the next page, now shown with numbers.

Tree for drive/bus example:




Calculate expected monetary values (weighted averages)

a .3 x (-12) = -3.6
.7 x (-2) = -1.4 added together -5.00

b .9 x (-5) = -4.5
.1 x (-6) = -.6 added together –5.10

c compare (b) to cost of garage and choose least costly

d [.8 x (-2.5)] + [.2 x (–5.1)] = -3.02

e compare (d) to cost of bus and choose least costly

Figure 6

First, for each branch, we add up the revenues (in this problem, there is no money coming in) and subtract the sum of the costs. So, DRIVE – no legal space – TRY ILLEGAL – space – ticket would indicate that at the end of the branch. If we follow the same path and are lucky enough to escape without a ticket, we're out only $2: so –2 appears after "no ticket."

Follow this procedure to put all the monetary information in the tree at the ends of all the branches. Then insert the probabilities over each of the possibilities at each circle. If we're looking at the completed tree, we then wonder how various numbers appear in the squares and circles. That's done by averaging and comparing.

To illustrate, take the branch that goes the farthest, ending in ticket/no ticket. The probability of getting a ticket is .3 with a monetary outcome of –12; thus there's a .7 chance of not getting a ticket with a monetary outcome of –2. To get a representative number for being in the position just before that circle, having parked illegally, we average the –12 and the –2. But it's not a straight average where one adds the numbers and divides by two – it's a weighted average since I'm less likely to get a ticket and end up with a cost of $12. How do we compute this kind of average? Multiply each of the monetary outcomes by the probability of getting that amount and add them all up. Here, it's .3*(-12) + .7*(-2) = -5; so over time, we would interpret that as an average of $5/day for parking illegally. We signify this by entering –5 in the circle as shown.

Backing down that branch, we can think of –5 as indicating the expected monetary value of parking illegally (EMV is used to express that). But what if I can't even find an illegal space, the other possibility associated with the choice of trying to park illegally?

There is only a 10% chance of that happening, so I was told. If it does happen, I will then head for the garage, so the monetary outcome = -(2, gas & tolls, + 4, parking) = -6. Now, continuing backward, we ask "what's the average cost of trying to park illegally?", focusing on the first circle. Do the same weighted averaging: .9 * (-5) + .1 * (-6) = -5.10.

Still working back along that branch and the ones it interfaces, I need to decide whether to go for the garage, as soon as I discover there are no legal spaces, or try to park illegally. If my decision is purely economically based, I end up comparing –6 for the garage with –5.10 for try illegal. There is no computation in the decision making. We put –5.10 in the box associated with "what to do if no legal space" because that is the optimal choice at that point.

Finally, we hook up to the space-no space on Beacon Hill, with .8 probability of space and a cost of $2.50 and a .2 probability of no space with an expected cost of $5.10. Multiplying and adding as indicated, we end up with –3.02 in the circle after DRIVE. That beats the bus at –3.40. So I should drive if all I care about is the bottom line.

If the stakes were higher as they are in business situations, this optimizing technique – called the EMV criterion – is utilized if two conditions hold:

1. The firm can afford the worst case scenario (solvency)
2. The firm faces many such situations and needs a general policy (repeatability)

Even in the case of my going to work, those two conditions hold. So I would use EMV if the situation called for monetary optimization alone.

### *Problem solving and the MUD list*

When one gets a problem to work on, all of a sudden it's not so direct. That's because when people present information, they do so for their ease of expression, not for ease of analysis. The R T C I approach will help.

I present the JM problem alluded to earlier in its entirety. "Johnson's Metal (JM), a small manufacturer of metal parts, is attempting to decide whether or not to enter the competition to be a supplier of transmission housings for PROTRAC. In order to compete, the firm must design a test fixture for the production process and produce 10 housings that PROTRAC will test. The cost of development, that is, designing and building the test fixture and the test housings, is $50,000. If JM gets the order, an event estimated as occurring with probability 0.4, it will be possible to sell 10,000 items to PROTRAC for $50 each. If JM does not get the order, the development cost is essentially lost. In order to produce the housings, JM may either use its current machines or purchase a new forge. Tooling with the current machines will cost $40,000 and the per unit production cost is $20. However, if JM uses its current machines, it runs the risk of incurring overtime cost. The relationship between overtime cost and the status of JM's other business is presented in Figure 15.39. The new forge costs $260,000, including tooling costs for the transmission housings. However, with the new forge, JM would certainly not incur any overtime costs, and the production cost will be only $10 per unit. Use a decision tree to determine the optimal set of actions for JM."

Figure 15.39

Cost and Probability Data for Johnson's Metal Problem

| OTHER BUSINESS | PROBABILITY | OVERTIME COST TO JM |
|---|---|---|
| Heavy | 0.2 | $200,000 |
| Normal | 0.7 | $100,000 |
| Light | 0.1 | 0 |

Students can do the first read through without benefit of the course or the framework: ask, who are the companies; for whom are we consulting? (active reading); what are the main issues without). This establishes JM as the company of record and its bidding/producing dilemma.

The second reading is just an overview of the issues the company faces: what are they, omitting details? (competing, getting, producing; the main one has to do with bidding: they don't know if they will get the contract and if they don't, they'll lose money because of the test apparatus they have to build as a condition for bidding.

The third read through uses the framework, with the MUD list (categorizing information bits as either "modeling," "uncertainty," or "decision," ignoring monetary outcomes and probabilities). The first sentence identifies a decision, possibly named "competition." The details may confuse, but the goal remains categorization – so we don't worry about a transmission housing! The second sentence tells us the conditions for competition. That's beyond JM's control, so it's not under the D column (not a decision), and it's definite, so it's not under the U column: here we have an example of modeling. The status of the bid is identified as an uncertainty as is the status of JM's other business.  The choice of technology is seen as a decision, since it's up to them.  Merely identifying these pieces poses a challenge for many students even if made explicit through the list mechanism, in turn prompted by effective reading strategies.  Trying to proceed without systematically reading to identify them quickly becomes a source of confusion.

Once students compile the list, they realize that for all the details, it contains just two decisions and two uncertainties.  The next read through involves, again non-quantitatively, specifying the choices (associated with decisions) and the possibilities (associated with uncertainties).  Students need to prompt themselves to note that JM can either enter or not, can use current machines or a new forge (to produce), will get the order or not, and will have heavy, medium, or light other business.

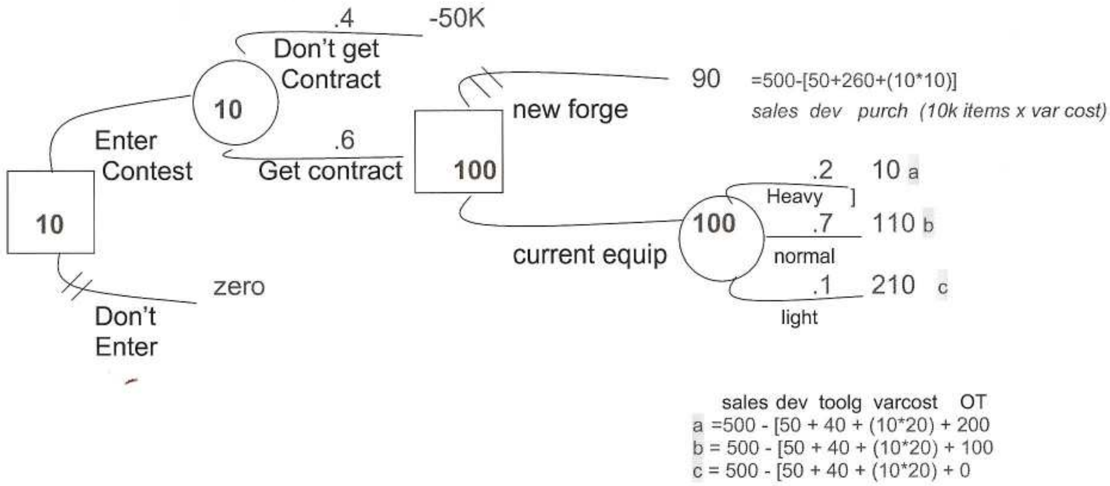
**The MUD list**:

| | |
|---|---|
| Bidding on a contract | D: (enter competition, don't enter) |
| In order to compete, test fixture | M: (condition of entering, imposed on them) |
| Getting the order | U: (get it, don't get it) |
| Choice of technology if get it | D: (new forge, current machines) |
| Volume of other business | U: (heavy, normal, light) |
| No overtime cost w/new forge | M: (known) |

Finally, it's time for the decision tree, the first "technical" order of business.  Here again, it proves helpful to build the tree with no numbers, referring only to the list.  The tree (as framework) simply shows chronologically all the relationships between the decisions and uncertainties. (The students who act on the information prematurely try to construct the tree as they read, putting in all the numerical details at the same time; or they simply list their notes in the order presented, borrowing the tree format – the squares, circles, and lines  In this case, they try to graft private reasoning onto a public format).

Now, students should read one more time, selectively for the numbers as requested by the tree.  At the end of each branch, they will put the sum of all the revenues accrued along that pathway minus the sum of all the costs incurred along that pathway.  They will have created slots for the relevant quantitative information: the tree "asks" them for these numbers, ready to receive them.  Any computations fit into this framework (such as unit cost times number of items produced).  No heroic quantitative effort is required.

Figure
7




This actually includes all of the last three stages. The tree itself with designations (enter contest, etc.) represents the translation. The numbers and their manipulation constitute the computation stage. This includes several steps:

1. The numbers at the ends of the branches computed by adding all the revenues along the branch and then subtracting the sum of the costs incurred along it. So entering and not getting the contract is a loss of $50,000 since they put that amount into building the test fixture. The other sums are explained by the words above or below the numbers: 110 for the branch ending in normal other business comes from the revenues (100 units at $50 each = $500,000 = 500 K); there are 4 costs: the development cost of the test fixture, the tooling cost, the cost of production based on $20 per unit and 100 units, and the overtime cost.
2. The averaging – comparing process, beginning at the ends of the branches and working backward So $100,000 is the expected value of using the current equipment, computed as (.2)(10) + (.7)(110) + (.1)(210). Comparing that to $90,000 for the new forge, the status quo prevails. And on it goes.

**WHAT STUDENTS CAN DO TO HELP AVOID ANXIETY**

Students can learn to be proactive and evaluate how the instructor is performing in a reasonably objective manner, provided they know what to look for. To start, they can see how instructors R.A.T.E.D., where now R stands for Resource customization, diversity, and flexibility, speaking to whether the instructor teaches mainly to the group or also focuses on individuals; A stands for awareness of student feelings and learning issues; T stands for willingness to talk about these issues, including his or her comfort level with that kind of conversation. E stands for empathic and finally D stands for dedication to and discipline in his/her commitment to good teaching.

This evaluation of instructor by students is not new. CEIs have been around for years on an institutional level; however, they do not help current students for they are generally gathered at the end of the semester and are really geared for evaluative purposes in helping department heads and tenure committees make decisions.  Actually, students are providing a service to the Administration.

What I have in mind is far different.  Students should use it privately and informally whenever they begin to get down on themselves and start concluding that it's all their fault – or at any point as a spot check.  This "questionnaire" is designed to help them get out from under theoretical abuse and the possible yoke of instructor tyranny and transference power.

But it doesn't stop there.  Students can use the "results" to ask for what they think is missing: They can ask questions about instructor processes and evaluate his/her response: is it an open one, a defensive one, or a thoughtful one?  For the latter, the instructor needs to think about it while for the former, s/he can come up with the information right away.

What sorts of questions does the student need to ask?  The main concerns are focus, prompts, and transitions – for these are the crucial typically concealed micro maneuvers that allow instructors to move around the discipline with ease.  Can they be transferred to students?  In their hands, would they be as powerful?   Sankowsky (2001) urges professors to teach poor performing students expert strategies.   Even if we're not sure they will work, it's worth trying, especially if current student strategies have proven ineffective.

For students who claim that math is their one (and only) stumbling block, this should be very helpful – especially for those who excel at learning languages and those studying linguistics, a discipline that is meta to languages.  These students can work the language angle, learning about existential and universal quantifiers to help them understood the purpose of two types of equations and three types of symbolic expression.  Non quantitative parallels should also help make these definitions and applications even more powerful,

For instance, a universal statement in English concerning numbers is "Adding zero doesn't change anything" comes out as x + 0 = x for all x € R, where the "epsilon" means is a member of, and R stands for the set of real numbers.  The notion of x as a placeholder for numbers is crucial.

Should the instructor rate low on one or more of these dimensions, students will not benefit much in merely shifting the blame for their poor performance.  Rather, they need to help themselves and their instructors get better. In this case, they can take the lead.

By expressing interest in the instructor's process, students will be able to ask "what did you ask yourself?", not just "what did you do?":  and "what did you focus on?,"  not just "what does it mean?"  If this instructor is unwilling or unable to divulge this information, students can ask other people, including high achievement peers.

Structural concerns are not new to students.  They show up in metaphors and in poetry in general. For a structural connection between A and B is any variable other than content that links them. Transitions in structure should be heralded, especially what had been a relationship turns into an individual.  They have seen that as far back as their first number learning.  For example, the number 3 is first a relationship linking 3 chairs, 3 SVUs, and 3 random thoughts: the "threeness" brings these individuals together.  3 brings them together but it's always 3 apples, for example, or some other specific object.

There are two issues here: one is the link between individuals; the *relationship* between 3 apples and 3 oranges is a connection, specifically, a one to one correspondence between the apples and the oranges using the counting variable to forge this link (And whoever said you couldn't compare apples and oranges?!). The second issue is that the number 3 (by itself) is an abstraction, on another level, where it is also now an *individual*, one of many numbers. The transition from 3 as relationship to 3 as individual is the earliest example I can think of. Students need to have this make explicit so that they realize they've been there before.

In general, students can help themselves by tuning into structural issues, even when their instructors do not. And that brings me to the central question for any writing: for whom is it intended, i.e. who's the audience. Actually, I thought it would be instructors of the courses in question: but it's also for students so that they can become aware of the issues. This page and a half addendum is the only part written exclusively for students, because most of the action is in the hands of instructors.

**Conclusion**

I have attempted to make a case for remedial and preventative approaches to teaching quantitative methods courses at the college level. My personal realization that math anxiety is alive and well even at the best schools is reinforced by others (Freiberg, 2005; Guillaume & Kirtman, 2005; Walker & Karp, 2005). I have attempted to fashion explanations for this phenomenon that are dynamic – as opposed to such static observations that more females suffer from it than males (Hyde, Fennema, & Lamon, 1990) – and present-based, treating the problem as an ongoing one, in spite of or in addition to its past roots. I single out a new paradigm, the development-based paradigm, that can be operationalized so that the adult learner may have immediate access to information that helps him or her surface and identify suffocating assumptions and unproductive tendencies. This will enable instructors to better intervene in allowing him or her to substitute a more strategic action in context. Learning takes place for both students and instructors, according to this model; the motto for instructors then is LEARN, SHARE, & REVEAL, to undo the prior damage and to prevent any additional angst.

## REFERENCES


Alderfer,C. (1993). An intergroup perspective on group dynamics. In J.Lorsch (Ed.) **Handbook of organizational behavior**. Englewood Cliffs, NJ: Prentice-Hall.

Arnkoff, D. (1980). Psychotherapy from the perspective of cognitive theory. In M. Mahoney (Ed.) **Psychotherapy process**, 339-361.

Argyris, C. (1990) **Overcoming organizational defenses: facilitating organizational change**. Boston: Allyn & Bacon

Ashcraft, M. & Faust, M. (1994). Mathematics anxiety and mental arithmetic performance: An exploratory investigation. **Cognition and emotion**, 8(2),97-125

Ausubel, D.P. 1968. **Educational psychology: A *c*ognitive view**. New York: Holt, Reinhart, & Winston.

Barnes, M. (1984) Understanding maths anxiety. **Vinculum**, 21(2), 14-19

Bedrosian, R. & Beck, A. (1980) Principles of cognitive therapy. In M. Mahoney (ed.) **Psychotherapy Process**, pp. 127-152. New York: Plenum

Dossel, S. (1993) Mathematics anxiety. **The Mathematics Teacher**,85(4), 298-299.

Eppen, G., Gould, F.J. & Schmidt, C. (1996). **Introductory management science.** Englewood Cliffs, NJ: Prentice Hall.

Freiberg, M. (2005) Math – that four-letter word! **Rapid Intellect Group**, Inc.

Freud, S. (1959). **Group psychology and the analysis of the ego**. New York: Norton (original work published 1921)

Gersick, C. (1991) Revolutionary change theories: A multilevel exploration of the Punctuated equilibrium paradigm. **Academy of Management Review,** 6(1), 10-36.

Goleman, D. (1996). **Emotional intelligence**. New York: Bantam

Greenwood, J. (1984) My anxieties about math anxiety. **Mathematics teacher**, 77: 662-663.

Guillaume, A.M. & Kirtman, L. (2005) Learning lessons about lessons: **Memories of mathematics instruction**, 11, 302-309.

Hornby, G. (1990). A humanistic developmental model of counseling: a psycho-educational approach. **Counselling Psychological Quarterly**. 3, 191-203.

Hyde, J.S., Fennema, E., & Lamon, S.J. (1990). Gender differences in mathematics performance: A meta-analysis. **Psychological Bulletin**, 107, 139-155.

Kahn, W. 1996. Comment on “Understanding researcher ‘projection’ in interpreting case study data: The South Canyon fire tragedy.” *Journal of* **Applied Behavioral Science**, 32, 62-69.

Lowenstein, J., Thompson, L. & Gentner, D. (2003) Analogical learning in negotiation teams: Comparing cases promotes learning and transfer. **Academy of Management Learning and Education**, 2(2), 119-127.

Mitchell, C.E. (1987) **Math anxiety**: **what it is and what to do about it**. Tempe: Action.

Newell, A. & Simon, H.A. (1972). **Human problem-solving.** Englewood Cliffs, NJ: Prentice-Hall.

Norwood, K. (1984) The effect of instructional approach on math anxiety and achievement. **School science and mathematics**, 94(5): 248-254.

Owen, E., & Sweller, J. 1989. Should problem solving be used as a learning device in mathematics? **Journal of Research in Mathematics Education**, 20, 322-328.

Rentsch, J.R., Heffner, T.S., & Duffy, L.T. 1994. What you know is what you get from experience. **Group & Organization Management**, 19(4), 450-474.

Resnick, L. (1983). Mathematics and science learning: a new conception. **Science**, 224, 477-478.

Rittle-Johnson, B., Siegler, R.S., & Alibali, M.W. 2001. Developing conceptual understanding and procedural skill in mathematics: An iterative process. **Journal of Educational** *Psychology*, 93(2), 346-362.

Sankowsky, D. & Ornstein, S. (1990). A process model of humor: Bringing SPAM into the classroom. **The Organizational Behavior Teaching Review**

Sankowsky, D. (1998). Two pathways to instructional delivery: distinguishing between the discipline-based and the development-based paradigms. **Journal of Management Education**, 22(3), 269-284.

Sankowsky, D. (2003). Problem-based learning: A modified approach for introductory management science. **Southwestern Business Administration Journal**, 3(1), 94-102.

Scardamalia, M. & Bereiter, C. (1991). Literate expertise. In K.A. Ericcson & J. Smith (eds.), **Toward a general theory of expertise**. Hillsdale, NJ: Lawrence Erlbaum (pp. 172-194).

Schoenfeld, A.H. 1989. Cognitive science and mathematics education: An overview. In A.H. Schoenfeld (Ed.), **Cognitive Science and Mathematics Education:** 1-32. Hillsdale, NJ: Lawrence Erlbaum.

Seligman, M.E.P. (1992). **Learned Optimism*: how to change your mind and change your life**. NY: Simon & Schuster.

Stuart, V. (2000) Math curse or math anxiety? **Teaching children mathematics**, 6(5); 330-335.

Tobias, S, (1993) **Overcoming math anxiety: revised and expanded.** New York: Norton.

Treisman, U. 1992. Studying students studying calculus: A look at the lives of minority mathematics students in college. **The College Mathematical Journal**, 23(5), 362-372.

Turner, J.C., Midgley, C., Meyer, D.K., Gheen, M., Anderman, E.M., Kang, Y., & Patrick, H. 2002. The classroom environment and students' reports of avoidance strategies in mathematics: A multimethod study. **Journal of Educational Psychology***, 94*(1), 88-106.

Vacc, N.N. (1993) Teaching and learning mathematics through classroom discussion. **Arithmetic teacher**, 41: 225-227.

Walker, E.N. & Karp, A.P. **Education Week** 24 (39)